TPK4120 - Lecture summary

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Updated 2021-09-22

Chapter 6 - System Reliability Analysis

Up to now we have mainly used the symbol x_i to represent the value a state variable may take. In order to assess component and system reliability, we need to treat the state variables and the structure function as random quantities (stochastic variables). We let $X_i(t)$ denote the state variable i, and $\mathbf{X}(t) = (X_1(t), X_2(t), ..., X_n(t))$ be the state vector. Further the structure function is now a random quantity, i.e., $\phi(\mathbf{X}(t))$. Now, introduce the following probabilities:

$$\begin{aligned} p_i(t) &= \Pr(X_i(t) = 1) = \text{ Component reliability} \\ p_S(t) &= \Pr(\phi(\mathbf{X}(t)) = 1) = \text{ System reliability} \end{aligned}$$

Since both the state variables and the structure function is binary we have:

$$\begin{aligned} \mathrm{E}(X_i(t)) &= p_i(t) \\ \mathrm{E}(\phi(\mathbf{X}(t))) &= p_S(t) \end{aligned}$$

Since the system reliability $p_S(t)$ depends on the component reliabilities, we often write:

$$p_S(t) = h[p_1(t), p_2(t), \ldots, p_n(t)] = h[\mathbf{p}(t)]$$

Reliability of series structures

Since the structure function of a series structure is the product of the state variables, we have

$$h[\mathbf{p}(t)] = \mathrm{E}(\phi(\mathbf{X}(t))) = \mathrm{E}\left(\prod_{i=1}^{n} X_i(t)\right) = \prod_{i=1}^{n} \mathrm{E}[X_i(t)] = \prod_{i=1}^{n} p_i(t)$$

where we have used that the expected value of a product equals the product of the expectations *if* the stochastic variables are *independent*.

Reliability of parallel structures

A similar argument may be used for a parallel structure which gives:

$$h[\mathbf{p}(t)] = 1 - \prod_{i=1}^{n} [1 - p_i(t)] = \coprod_{i=1}^{n} p_i(t)$$

In a more general setting assume that we are able to write the structure function as a *sum of products* of the state variables. Further assume *independent* components and that we have removed any exponents in the expressions, i.e., $x_i^n = x_i$. We then use the results that "the expectation of a sum equals to the sum of expectations" and "the expectation of a product equals the product of the expectations". This means that $p_S(t) = \mathrm{E}(\phi(\mathbf{X}(t)))$ will equal the sum of products of expectations, i.e., a sum of products of $\mathrm{E}(X_i(t))$'s. Further since $\mathrm{E}(X_i(t)) = p_i(t)$ we have proven that the system reliability $p_S(t)$ may be found by replacing all the x_i''s in the structure function with corresponding $p_i(t)$'s.

Note that this approach is only valid if we have carried out the multiplication, i.e., resolved any parentheses in the expression for the structure function, and removed any exponents.

General approach utilizing the structure function

1. Map the physical system into a reliability block diagram or another representation as a starting point
2. Use various approaches (series, parallels, bridges, k-out-of-n's etc) to derive the structure function
3. Multiply out any parentheses, collect terms, and remove any exponents, yielding a structure function as a sum of products
4. The system reliability, $p_S(t)$ is now found by replacing all the x_i's with corresponding $p_i(t)$'s in the sum of product version of the structure function

Example

Assume that we have component 1 in series with a parallel of two components 2 and 3 as shown in Figure 1

The structure function is

$$\phi(\mathbf{x}) = x_1 \cdot (x_2 \amalg x_3) = x_1x_2 + x_1x_3 - x_1x_2x_3$$

We now replace all the x_i''s in the structure function with corresponding $p_i(t)$'s to get the system reliability. Assuming $p_1 = 0.99$, $p_2 = p_3 = 0.9$ gives:

$$p_S(t) = p_1p_2 + p_1p_3 - p_1p_2p_3 = 0.891 + 0.891 - 0.8019 = 0.9801$$

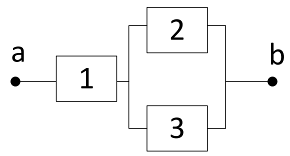

Figure 1: Example RBD

Pivotal Decomposition and Critical component

In connection with bridge structure in chapter 4 we introduced the technique of pivotal decomposition, i.e.,

$$\phi[\mathbf{X}(t)] = X_i(t)\phi[1_i,\mathbf{X}(t)] + [1 - X_i(t)]\phi[0_i,\mathbf{X}(t)]$$

where 1_i and 0_i are flags to specify that component i takes the values 1 and 0 respectively. Since we assume independent components we use the product rule for expectations, and by rearranging the terms we get:

$$h[\mathbf{p}(t)] = \mathrm{E}(\phi(\mathbf{X}(t))) = p_i(t)\left(h\left[1_i,\mathbf{p}(t)\right] - h\left[0_i,\mathbf{p}(t)\right]\right) + h\left[0_i,\mathbf{p}(t)\right]$$

where 1_i and 0_i are flags to specify that reliability of component i is 1 and 0 respectively

Component i is said to be critical if the rest of the components are in such states that the system is functioning when component i is functioning and fails when component i fails. Thus component i is critical if and only if $\phi[1_i,\mathbf{X}(t)] = 1$ and $\phi[0_i,\mathbf{X}(t)] = 0$. For a coherent and binary system component i is critical if and only if:

$$\phi[1_i,\mathbf{X}(t)] - \phi[0_i,\mathbf{X}(t)] = 1$$

The probability that component i is critical is thus given by:

$$\Pr(\text{Component } i \text{ is critical}) = \Pr(\phi[1_i,\mathbf{X}(t)] - \phi[0_i,\mathbf{X}(t)] = 1)$$
$$\mathrm{E}\left(\phi[1_i,\mathbf{X}(t)] - \phi[0_i,\mathbf{X}(t)]\right) = h\left[1_i,\mathbf{p}(t)\right] - h\left[0_i,\mathbf{p}(t)\right]$$

where we have used that the $[1_i,\mathbf{X}(t)] - \phi[0_i,\mathbf{X}(t)]$ is binary to go from probability to expectation. If this probability is high, component i has a high reliability importance. This is discussed in Chapter 7.

Non-repairable Systems

Up to now we have been considering the instant reliability at a given point of time, say t. $p_S(t)$ is then the probability that the system is functioning at time t. For non-repairable systems we may use the same argument also to find the probability that the system survives time t by inserting component survival probabilities in the structure function as we did with the instant probabilities. This will not be the case for repairable systems because a system may survive t even though some or all of the components have failed, if they are repaired and we are saved by redundancy.

Non-repairable Series System

$$R_S(t) = \prod_{i=1}^{n} R_i(t)$$

where $R_S(t)$ is the survivor function of the system, and $R_i(t)$ is the survivor function of component i. By using the relation between the survivor function and the failure rate function we obtain:

$$R_S(t) = e^{-\int_0^t \sum_{i=1}^n z_i(u)du}$$

where $z_i(t)$ is the failure rate function of component i. Hence for a series structure the failure rate function is a sum of the failure rate functions of the components:

$$z_S(t) = \sum_{i=1}^{n} z_i(t)$$

Non-repairable Parallel System

$$R_S(t) = 1 - \prod_{i=1}^{n} (1 - R_i(t))$$

The formula for e.g., the failure rate function and the MTTF is difficult to obtain in the general case. Some results may be derived in special cases. For example for two components of the same type with constant failure rate λ we have:

$$R_S(t) = 2e^{-\lambda t} - e^{-2\lambda t}, \quad \text{MTTF} = \frac{3}{2\lambda}$$

Nonrepairable 2-out-of-3 system

Three components of the same type with constant failure rate λ

$$R_S(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t}, \quad \text{MTTF} = \frac{5}{6\lambda}$$

Non-repairable k-out-of-n system

For a k-out-of-n system it is hard to obtain general results, but for n components of the same type with constant failure rate λ we may obtain:

$$R_S(t) = \sum_{x=k}^{n} \binom{n}{x} e^{-\lambda t x}(1 - e^{-\lambda t})^{n-x}$$

$$\text{MTTF} = \frac{1}{\lambda} \sum_{x=k}^{n} \frac{1}{x}$$

The MTTFs of some k-out-of-n systems of identical and independent components with constant failure rate λ are listed in Table 1

Table 1: MTTF of some k-out-of-n Systems of Identical and Independent Components with Constant Failure Rate λ.

$k \backslash n$	1	2	3	4	5
1	$\frac{1}{\lambda}$	$\frac{3}{2\lambda}$	$\frac{11}{6\lambda}$	$\frac{25}{12\lambda}$	$\frac{137}{60\lambda}$
2	–	$\frac{1}{2\lambda}$	$\frac{5}{6\lambda}$	$\frac{13}{12\lambda}$	$\frac{77}{60\lambda}$
3	–	–	$\frac{1}{3\lambda}$	$\frac{7}{12\lambda}$	$\frac{47}{60\lambda}$
4	–	–	–	$\frac{1}{4\lambda}$	$\frac{9}{20\lambda}$
5	–	–	–	–	$\frac{1}{5\lambda}$

Single repairable items

Recall the state variable of an item:

$$X(t) = \begin{cases} 1 \text{ if the item is functioning at time } t \\ 0 \text{ if the item is in a fault state at time } t \end{cases}$$

The availability $A(t)$ of an repairable item is given by:

$$A(t) = \Pr(X(t) = 1) = \Pr(\text{The item is in a functioning state at } t)$$

Unavailability is given by:

$$U(t) = \overline{A}(t) = 1 - A(t) = \Pr(X(t) = 0) = \Pr(\text{The item is in a failed state at } t)$$

Since availability is time dependent, we would in some cases be interested in the average availability in a time interval (t_1, t_2), i.e., the interval availability:

$$A_{\text{avg}}(t_1, t_2) = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} A(t)dt$$

The limiting availability is given by:

$$A = \lim_{t \to \infty} A(t)$$

Average Availability with Perfect Repair

Consider a component which is repaired after a failure. Assume that failures and repairs are independent with constant mean up times (MUT) and mean down times (MDT).

For a large number of periods where each period is covering the uptime and the downtime. The total percentage of uptime is given by

$$A = \frac{\sum \text{uptimes}}{\sum \text{uptimes} + \sum \text{downtimes}} = \frac{\text{average uptimes}}{\text{average uptimes} + \text{average downtimes}}$$

In the limit the average values converges to the expected values, hence

$$A_{\text{avg}} = \frac{\text{MUT}}{\text{MUT} + \text{MDT}}$$

Note that we have introduced the term MUT = Mean Up Time rather than the more familiar term MTTF = Mean Time To Failure. The reason for this is that MTTF is always mean time to first failure for an item which is considered to be perfect at time t=0. In general repairs will not put the item back to a perfect state and the term MUT is used for the mean up times if this mean value exist. In many textbooks we rather see the formula $A_{\text{avg}} = \text{MTTF}/(\text{MTTF} + \text{MDT})$.

In chapter 11 is is shown that if repair times and failure times are exponentially distributed with rates μ = 1/MDT and λ = 1/MUT respectively the availability is given by:

$$A(t) = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda+\mu)t}$$

from which it follows that

$$\overline{A} = \frac{\text{MDT}}{\text{MUT} + \text{MDT}}$$

If MDT << MUT this formula simplifies

$$\overline{A} = \lambda \text{MDT}$$

which is a formula used in hand calculations.

ROCOF = Rate of OCcurence Of Failures

The failure rate function $z(t)$ was introduced to describe the conditional probability of failure for an item which has not experienced any failures yet. For repairable items this first uptime interval is not that relevant and the ROCOF is therefore introduced. To define the ROCOF we need to have a stochastic process perspective, i.e., we consider what is happening in a time interval rather when things are happening in this interval. Let $N(t)$ be the number of failures that occur in $(0, t]$ and let $W(t) = \text{E}[N(t)]$. The ROCOF at time t is now defined by

$$w(t) = \lim_{\Delta t \to 0} \frac{\text{E}[N(t + \Delta t) - N(t)]}{\Delta t} = \lim_{\Delta t \to 0} \frac{W(t + \Delta t) - W(t)}{\Delta t} = \frac{d}{dt} W(t)$$

Consider an item which is repaired to as good as new after each failure. The average length of a failure/repair "cycle" is then MUT + MDT, hence the expected number of failures per time unit will asymptotically approach:

$$w = \frac{1}{\text{MUT} + \text{MDT}}$$

Repairable systems

We have seen that the structure function is the basis for calculating system reliability. In the same way as we did for system reliability we obtain system *availability* by replacing all the x_i's with the corresponding component availabilities $A_i = \text{MUT}_i/(\text{MUT}_i + \text{MDT}_i)$. In the following we will also obtain the ROCOF, MUT and MDT at system level. We use similar notation as previously but now we replace "reliability" with "availability", where $\mathbf{A}$ is a vector of the component availabilities.

ROCOF for repairable systems

A system failure "caused" by component i will occur if:

- Component i is critical ("the other components")
- Component i is functioning, and then
- Component i fails

The probability that component i is critical was found to be $h(1_i, \mathbf{A}) - h(0_i, \mathbf{A})$ and the failure frequency (ROCOF) of component i was found to be $w_i = 1/(\text{MUT}_i + \text{MDT}_i)$. The ROCOF could be used as an approximation for the combination of the component is functioning, and then failing, but we can be more explicit by multiplying the component failure rate with the component availability If failure times and repair times are exponentially distributed with rates λ_i and μ_i respectively the system failure frequency caused by component i is then given by:

$$w_S^{(i)} = \lambda_i \frac{\mu_i}{\lambda_i + \mu_i} [h(1_i, \mathbf{A}) - h(0_i, \mathbf{A})]$$

Summing over all components gives the total system failure frequency:

$$w_S = \sum_{i=1}^{n} \frac{\lambda_i \mu_i}{\lambda_i + \mu_i} [h(1_i, \mathbf{A}) - h(0_i, \mathbf{A})]$$

MUT and MDT for repairable systems

It follows from $A_S = h(\mathbf{A}) = \text{MUT}_S/(\text{MUT}_S + \text{MDT}_S)$ and $w_S = 1/(\text{MUT}_S + \text{MDT}_S)$ that mean system uptimes and downtimes are given by:

$$\text{MUT}_S = \frac{A_S}{w_S}$$
$$\text{MDT}_S = \frac{[1 - A_S]\text{MUT}_S}{A_S}$$

respectively.

Fault tree analysis

Introduce

$$Y_i(t) = \begin{cases} 1 \text{ if basic event } i \text{ occurs at time } t \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

Further introduce the structure function of the fault tree, $\psi(\mathbf{Y}(t))$:

$$\psi(\mathbf{Y}(t)) = \begin{cases} 1 \text{ if the TOP event occurs at time } t \\ 0 \text{ otherwise} \end{cases}$$

As for the reliability block diagram we are working with binary variables, hence

$$q_i(t) = \text{E}(Y_i(t)) = \text{Pr}(Y_i(t) = 1) = \text{ Basic event probability}$$
$$Q_0(t) = \text{E}(\psi(\mathbf{Y}(t)) = \text{Pr}(\psi(\mathbf{Y}(t)) = 1) = \text{ TOP event probability}$$

Fault tree with a single AND-gate

Consider a fault tree with n basic events under a single AND-gate. Since an AND-gate requires all basic events to occur ($y_i(t) = 1$), the structure function is:

$$\psi(\mathbf{Y}(t)) = Y_1(t) \cdot Y_2(t) \cdot \ldots \cdot Y_n(t) = \prod_{i=1}^{n} Y_i(t)$$

hence

$$Q_0(t) = \text{E}(\psi(\mathbf{Y}(t)) = \text{E}\left(\prod_{i=1}^{n} Y_i(t)\right) = \prod_{i=1}^{n} \text{E}[Y_i(t)] = \prod_{i=1}^{n} q_i(t)$$

if the basic events are independent.

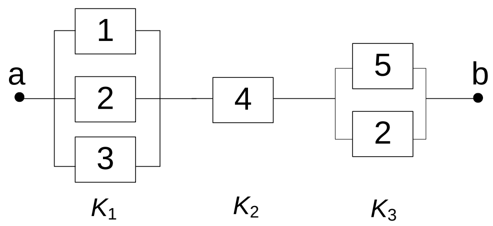

Figure 2: Example cut set structure

Upper Bound Approximation, $Q_0(t)$

Assume that we have found the minimal cut sets of the fault tree, i.e., K_j. Further assume that the minimal cut sets do not contain common components, hence they are independent (also provided that the components are independent). We may now arrange the cut set in a series structure as indicated in Figure 2: Let E_j denote the event that cut set number j is occurring. The probability that cut set number j is occurring is found by:

$$\Pr(E_j) = \check{Q}_j(t) = \prod_{i \in K_j} q_i(t)$$

We now have

$$\begin{aligned} Q_0(t) = \Pr(\text{TOP event occurs at time } t) &= 1 - \Pr(\text{TOP event does not occur at time } t) \\ &= 1 - \Pr(\text{No cut set occurs at time } t) \end{aligned}$$

Since the cut sets are independent, and the probability that cut set number j is occurring is given by $\check{Q}_j(t)$, we have:

$$Q_0(t) = 1 - \prod_{j=1}^{k} (1 - \check{Q}_j(t))$$

where

$$\check{Q}_j(t) = \prod_{i \in K_j} q_i(t)$$

Generally there might be some basic events that occur in two or more cut sets, hence the cut sets are *dependent*, and it may be proven that the formula represents an upper bound for the TOP event probability:

$$Q_0(t) \leq 1 - \prod_{j=1}^{k} (1 - \check{Q}_j(t))$$

Hence, we may use:

$$Q_0(t) \approx 1 - \prod_{j=1}^{k} (1 - \check{Q}_j(t))$$

which is referred to as the upper bound approximation and is usually considered to be a good approximation when the $q_i(t)$s are small.

To argue for the less or equal sign we realize that cut sets are "positive dependent" if they have common components. For two cut sets we have

$$\Pr(E_1^C \cap E_2^C) = \Pr(E_1^C|E_2^C)\Pr(E_2^C) > \Pr(E_1^C)\Pr(E_2^C)$$

and

$$Q_0 = 1 - \Pr(E_1^C \cap E_2^C) < 1 - \Pr(E_1^C)\Pr(E_2^C) = 1 - (1 - \check{Q}_1)(1 - \check{Q}_2)$$

and we may give similar arguments for more two or more cut sets.

The Inclusion-Exclusion Principle

Referring to Figure 2 it is also obvious that we may write:

$$Q_0(t) = \Pr(\cup_j E_j)$$

A challenge here is to find the probability of the union of events. For two events A and B we have $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$. For more than two events (cut sets) this becomes more complicated, and we have to use the general addition theorem in probability:

$$Q_0(t) = \Pr(\cup_j E_j) = \sum_j \Pr(E_j) - \sum_{i<j} \Pr(E_i \cap E_j) + \sum_{i<j<k} \Pr(E_i \cap E_j \cap E_k) - \ldots$$

To find $\Pr(E_i \cap E_j)$, $\Pr(E_i \cap E_j \cap E_k)$ is straight forward since these intersections of events are in fact intersection of a set of basic events, and we may multiply the corresponding probabilities as we have done for a single minimal cut set. The challenge is the number of terms we have to calculate. As a starting point we can only take the first sum, i.e., adding the cut set occurrences for each cut set. A slightly better approach would be to subtract the next sum. There are some ways we can optimize the calculations, and finding bounds for the answer to use as a stopping rule, see the textbook. Very often the inclusion-exclusion principle is used by only adding the cut set probabilities:

$$Q_0(t) \approx \sum_{j=1}^{k} \check{Q}_j(t)$$

which is faster than the upper bound approximation, but less accurate.

The next challenge is to find the basic event probabilities, $q_i(t)$. Three situations are often considered:

Non-repairable components

If a component cannot be repaired, the probability that it is in a fault state at time t equals $1-R(t)$, and provided that the component has an exponentially distributed life time, we therefore have:

$$q_i(t) = 1 - e^{-\lambda_i t}$$

where λ_i is the constant failure rate of the component.

Repairable components

To derive $q_i(t)$ for a repairable components we may use Markov analysis. The probability that the component is in a fault state at time t is then shown to be (according to eq. 8.22):

$$q_i(t) = \frac{\lambda_i}{\mu_i + \lambda_i}\left(1 - e^{-(\lambda_i+\mu_i)t}\right)$$

where λ_i is the constant failure rate of the component, and $\mu_i = 1/\mathrm{MTTR}_i$ is the constant repair rate. When t is large compared to $\frac{1}{\lambda_i+\mu_i}$ we have

$$q_i(t) \approx \frac{\lambda_i}{\mu_i + \lambda_i} \approx \lambda_i \mathrm{MTTR}_i$$

if repair times are short compared to failure times. If this holds, it is safe to use this approximation when $t > 3\mathrm{MTTR}_i$, where MTTR_i is the mean time to restoration for the component.

Periodically tested components

For components with a hidden function, it is usual to perform a functional test at fixed time intervals, say τ_i, to verify that the component is able to carry out it's function. In Chapter 10 it is shown that the (on demand) failure probability of such a component is given by:

$$q_i(t) \approx \lambda_i \tau_i / 2$$

TOP event frequency, w_{TOP}

w_{TOP} denotes the expected number of occurrences of the TOP event per unit time. The arguments are as follows:

- We know the minimal cut sets
- If one cut set should be the "contributor" to the TOP event to occur, the other cut sets cannot be occurring

- For a basic event in one cut set to bring the cut set to occur, requires that all other basic events in that cut set are occurring

Let $C_{\mathscr{K}}$ denote a minimal cut set, then the cut set occurrence frequency is given by:

$$\check{w}_{\mathscr{K}} = \sum_{i \in C_{\mathscr{K}}} w_i \prod_{\ell \in C_{\mathscr{K}}, \ell \neq i} q_\ell$$

where w_i is the ROCOF of basic event i, and q_l is the probability that basic event l is occurring.

To obtain the TOP event frequency we may now sum $\check{w}_{\mathscr{K}}$'s. However, note that $\check{w}_{\mathscr{K}}$ will not contribute to the TOP event frequency if one of the other cut set is already in a fault state, hence the TOP event frequency is better approximated by:

$$w_{\text{TOP}} \approx \sum_{\mathscr{K}=1}^{k} \check{w}_{\mathscr{K}} \prod_{j=1, j \neq \mathscr{K}}^{k} (1 - \check{Q}_j) \approx \sum_{\mathscr{K}=1}^{k} \check{w}_{\mathscr{K}} \frac{1 - Q_0}{1 - \check{Q}_{\mathscr{K}}}$$

Redundancy

The only model considered in the lecture was *Cold Standby*. Further we assume that all components are *non-repairable*. The aim of the modelling is to assess the survivor function, $R(t)$ of the system. The idea is now to split into disjoint ways the system survives t. One way to survive, say I, is obvious that there is no failure of the active item(s) up to time t. To find $R_{\text{I}}(t)$ in this situation should be straight forward. Another way the system may survive t, say II, is that (one of) the active item(s) fail at time τ, and then a cold stand by unit is activated. Then there should be no failures in the period from τ to t. In the modelling we need to take into account that the passive unit can be started. We can do this by using a fixed or time dependent probability, say $p(t)$, that represents success in activating the cold standby unit, or we may assume that the cold standby unit has a constant failure rate function in cold standby, and we may find the probability of successful start as the corresponding survivor function. Next we need to consider all possible τ-values in $[0, t]$ and integrate. This integral is typically given by:

$$R_{\text{II}} = \int_0^t p(\tau) f(\tau) R(t - \tau) d\tau$$

where $f(t)$ is the probability density function of the failure of the active item(s), $R(t)$ is the survivor function of the item(s) that has to keep the system functioning after the first failure, and $p(t)$ is the probability that we are able to activate the cold standby unit.

Example

Assume that the active and passive items have exponentially distributed life times with failure rates λ_1 and λ_2 respectively. Further assume that $p(t) = p$, i.e., constant over time. We have that $R_{\mathrm{I}} = e^{-\lambda_1 t}$, and:

$$R_{\mathrm{II}} = \int_0^t pf(\tau)R(t-\tau)d\tau = p\int_0^t \lambda_1 e^{-\lambda_1\tau}e^{-\lambda_2(t-\tau)}d\tau$$

$$= pe^{-\lambda_2 t}\int_0^t \lambda_1 e^{-(\lambda_1-\lambda_2)\tau}d\tau = \frac{p\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2 t} - \frac{p\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_1 t}$$

Hence, $R(t)$ is given by:

$$R(t) = e^{-\lambda_1 t} + \frac{p\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_2 t} - \frac{p\lambda_1}{\lambda_1-\lambda_2}e^{-\lambda_1 t}$$

If $p = 1$ and $\lambda_1 = \lambda_2 = \lambda$ we get:

$$R(t) = e^{-\lambda t} - \lambda t e^{-\lambda t}$$

which is recognized as the survivor function in the gamma distribution with parameters $k = 2$ and λ as expected. □

The procedure may be extended if there could be more than one failures in the interval considered. This will, however, be quite tedious.

Several examples are given in the textbook.

Main learning objectives: What to know from Ch 6

General approach utilizing the structure function

1. Map the physical system into a reliability block diagram or another representation as a starting point

2. Use various approaches (series, parallels, bridges, k-out-of-n's etc) to derive the structure function

3. Multiply out any parentheses, collect terms, and remove any exponents, yielding a structure function as a sum of products

4. Find component reliabilities, $p_i(t)$, depending on the situation, e.g., non-repairable, repairable, component periodically tested etc.

5. The system reliability, $p_S(t)$ is now found by replacing all the x_i's with corresponding $p_i(t)$'s in the sum of product version of the structure function.

General approach fault tree analysis

1. Define the TOP event by asking “what”, “where” and “when’
2. Identify boundary conditions
3. Draw the fault tree by starting from the TOP and search for direct causes, proceed until “basic events” are encountered
4. Find the minimal cut sets (MCS) by direct inspection or MOCUS
5. Analyse qualitative the MCS’s where MCS’s with low order are the most important ones
6. Find component failure probabilities, $q_i(t) = 1 - p_i(t)$, depending on the situation, e.g., non-repairable, repairable, component periodically tested etc.
7. The TOP-event probability, $Q_0(t)$ is found by the upper bound approximation, or simply adding the cut set contribution cut by cut.