TPK4120 - Lecture summary

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Chapter 8 - Dependent Failures

Two random quantities are independent if information regarding the value of one of them does not influence our probability distribution for the other one. Similarly, two events A_1 and A_2 are independent if $\Pr(A_1|A_2) = \Pr(A_1)$ and $\Pr(A_2|A_1) = \Pr(A_2)$. From this follows that $\Pr(A_1 \cap A_2) = \Pr(A_1)\Pr(A_2|A_1) = \Pr(A_1)\Pr(A_2)$. If $\Pr(A_1|A_2) > \Pr(A_1)$ there is a *positive* dependency between the two events. In this course we only consider positive dependency. In general dependent failures may be classified into three main groups:

1. Common cause failures (CCF). A common cause event is a dependent failure in which two or more component fault states exist simultaneously (or within a short time interval), and are a direct result of a shared cause.

2. Cascading failures. Cascading failures are multiple failures initiated by the failure of one component that result in a chain reaction or "domino effect".

3. Negative dependencies. Negative dependency failures are single failures that reduce the likelihood of failures of other components.

The β-Factor Model

Only one quantitative model is treated in the lecture, i.e., the β-factor model. The idea behind this model is to split the total failure rate of one component into an independent part and a dependent part:

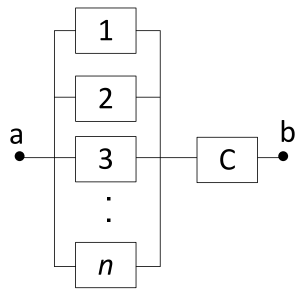
$$\lambda = \lambda^{(i)} + \lambda^{(c)}$$

We may think of the dependent part to be the part of the total failure rate that is caused by a common cause, i.e., a cause that causes all components to fail. In this model we do not model dependent failures explicitly, but rather we implicitly assume that a portion of the total failure rate is attributed to a common cause. This fraction is denoted the "common cause factor" given by:

$$\beta = \frac{\lambda^{(c)}}{\lambda}$$

yielding $\lambda^{(c)} = \beta\lambda$ and $\lambda^{(i)} = (1-\beta)\lambda$.

Now, consider a parallel structure of n components. In the base case we now assume that all components have the same constant failure rate λ with a common cause factor β. We now treat the common cause as a "virtual" components, say C. The reliability block diagram is the split into a parallel representing the "independent part", and this virtual component in series:



The "independent" components now each has a failure rate equal to $\lambda^{(i)} = (1-\beta)\lambda$, and the C-component has failure rate $\lambda^{(c)} = \beta\lambda$. We may now treat the RBD as a "normal" RBD of independent components and use results for series and parallel structures as in Chapter 4.

Markov example

Consider the example with the bike ride. At start-up, the bike is equipped with two functioning tyres. In addition we bring one spare tyre. Upon a puncture of one of the tyres, the failed tyre is replaced by the spare (in almost no time). First assume no common cause failure. We have the following system states:

- 3: Two functioning tyres on the bike, one functioning spare
- 2: Two functioning tyres on the bike, one failed tyre (not thrown away...)
- 1: One functioning tyre on the bike, one failed on the bike, and one failed tyre (not thrown away...)
- 0: Two failed tyres on the bike, one failed in addition

The Markov diagram is shown in Figure 1. Assuming CCF we modify the Markov diagram as shown in Figure 2.

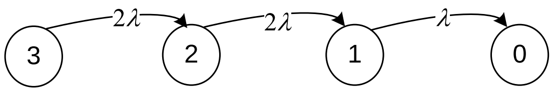


Figure 1: Markov diagram for the bike ride, independent failures

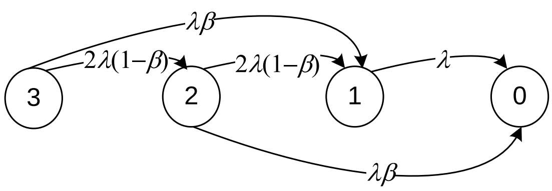


Figure 2: Markov diagram for the bike ride, common cause failures