# Finding minimal cut sets in combined event- and fault tree systems

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## 1 Introduction

This memo briefly describes how minimal cut sets may be obtained for combined event- and fault tree systems. It is assumed that the reader is familiar with the definition of cut sets in general, and how to obtain these.

## 2 Definitions:

*A **cut set** in a fault tree is a set of Basic events whose (simultaneous) occurrence ensures that the TOP event occurs. A cut set is said to be **minimal** if the set cannot be reduced without loosing its status as a cut set.*

*A **path set** in a fault tree is a set of Basic events whose non-occurrence (simultaneously) ensures that the TOP event does not occur. A path set is said to be **minimal** if the set cannot be reduced without loosing its status as a path set.*

For small and simple fault trees, it is feasible to identify the minimal cut- and path sets by inspection without any formal procedure/algorithm. For large or complex fault trees we need an efficient algorithm. The MOCUS algorithm (*Method for obtaining cut sets*) is described in standard FTA textbooks, and an efficient improvement of the algorithm is described by Vatn (1993).

**Dual fault tree**

Let FT be a fault tree with basic events $BE_i$. A dual fault tree to FT, say FT*, is obtained by changing all AND gates in FT to OR gates, and all OR gates in FT to AND gates, and finally the basic events of FT* are the complements of the corresponding basic events in FT.

**Theorem**

The minimal path sets of FT is given by the minimal cut sets of FT* with the complement basic events $BE_i$* are replaced with $BE_i$.

## 3 Approach

This result will be useful if we have implemented an algorithm to find minimal cut sets, and if we need the minimal path sets. When we combine event- and fault trees this will be the case.

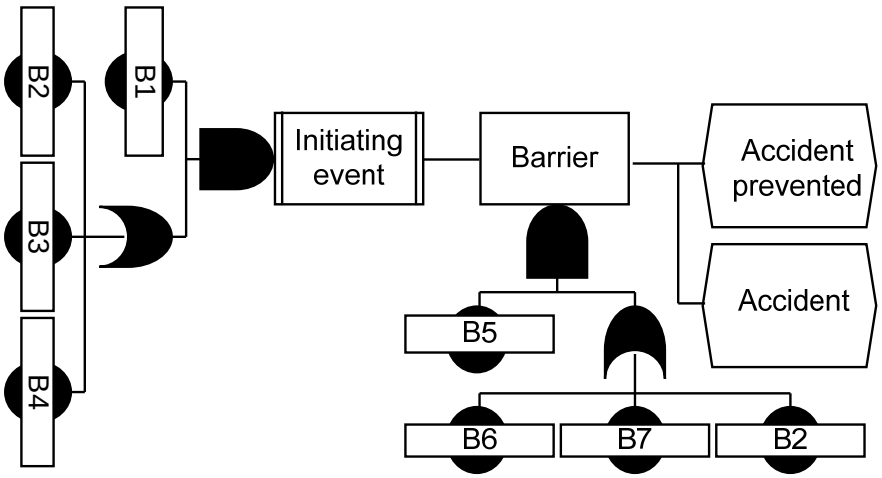


Figure 1 Example system combining event and fault tree

Figure 1 shows a system of combined fault and event trees. There is only one barrier in the event tree after the initiating event. To the left of the initiating event we have drawn a fault tree representing the combination of events that lead to the initiating event. Further below the barrier we have drawn a fault tree with the TOP event corresponding to the *failure* of the barrier. Two end consequences are drawn, one for "Accident prevented" corresponding to success of the barrier, and one for "Accident" corresponding to the failure of the barrier, i.e., the occurrence of the TOP event.

Minimal cut sets for the left most fault tree are given by {B1,B2},{B1,B3}, and {B1,B4}. Minimal cut sets for the fault tree below the barrier are given by {B5,B6},{B5,B7}, and {B5,B2}.

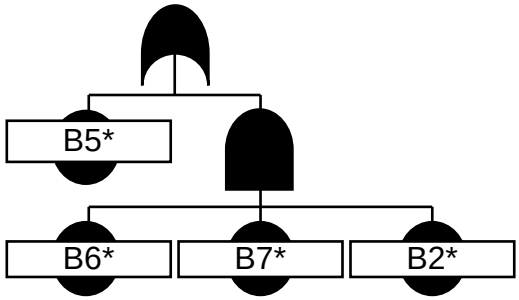


Figure 2 Dual fault tree

Figure 2 shows the dual fault tree of the rightmost fault tree in Figure 1. The minimal cut sets of this dual fault tree is {B5*} and {B6*,B7*,B2*}. Thus the minimal path sets of the original fault tree is given by {B5} and {B6,B7,B2}. We observe that the occurrence of at least one cut set of the dual fault tree will ensure that the outcome of the barrier is a success. For example the occurrence of {B5*} corresponds to the non-occurrence of basic event B5.

In order to find the cut sets for each end consequence in the combined fault- and event tree system we need two types of cut sets operators, the & operator, and reduction operators. Let CSs1 and CSs2 be two cut sets, where CSs1 contains the minimal cut sets CS11, CS12, ..,CS1m, and CSs2 contains the minimal cut sets CS21,CS22,..,CS2n. We now define the &-operator for two set of cut sets:

**&-operator**

The &-operator for two set of cut sets is defined such that CSs3 = CSs1 & CSs2 is a new set of cut sets where CSs3 is the set of all combination of minimal cut sets from CSs1 and CSs2. A combination of two cut sets CSa and CSb in this context is the set of all events in CSa and all events in CSb (the union of events in each of them).

Note that CSs3 might contain non-minimal cut sets. Also note that each cut set contained in CSs3 may contain repeated evens, and also events that may not occur simultaneously. After applying the &-operator we need post-processing of the result. Three types of reductions are necessary, (i) eliminate repeating events, (ii) remove cut sets with two or more events that may not occur simultaneously, and (iii) eliminate non minimal cut sets.

The idea for finding cut sets for each end consequence we collect relevant cut sets along the paths from the initiating event to the various end consequences. Note that if a fault tree is not developed for the initiating event or a barrier the cut sets only contain one cut set, and this one again is only one event, either (i) the initiating event, or (ii) the success, or (iii) failure of the barrier.

# 4 Procedure

The procedure for finding the minimal cut sets for one end consequence is now as follows where CSs is the running set of cut sets:

1. Start with the initiating event. If a fault tree is developed for the initiating event, let CSs be the corresponding set of minimal cut sets, else the set of cut sets is the initiating event it self.
2. Proceed with the next barrier along the path until the required end consequence is reached.
3. If the barrier along the path we are following corresponds to a failure of this barrier, let CSsB be the set of cut sets for the fault tree of the barrier, or let CSsB be the barrier failure it self if no fault tree is developed. If we are following the success of this barrier let CSsB be the minimal cut set of the dual fault tree, or if no fault tree exists for this barrier, let CSsB be the complement of the barrier failure, i.e., the barrier success.
4. Apply the & operator, i.e., CSs ← CSsB & CSs
5. Remove the second of repeated events in each cut set of CSs
6. Remove cut sets of CSs containing a basic event, say $B_i$, *and* it's complement $B_i$*.
7. Remove non-minimal cut sets in CSs
8. GoTo Step 2
9. When the end consequence is reached, CSs now is the set of minimal cut sets for this event

We now demonstrate the procedure for the example in Figure 1, and we start with the "accident" end consequence.

In Step 1 we find CSs = { {B1,B2},{B1,B3},{B1,B4} }. In Step 3 we find CSsB = { {B5,B6},{B5,B7},{B5,B2} }. By applying the & operator we get:

CSs = { {B1,B2,B5,B6}, {B1,B2,B5,B7}, {B1,B2,B5,B2},{B1,B3,B5,B6}, {B1,B3,B5,B7}, {B1,B3,B5,B2}, {B1,B4,B5,B6},{B1,B4, B5,B7},{B1,B4, B5,B2} }.

In Step 5 we remove one occurrence of B2 where it occurs twice, and get:

CSs = { {B1,B2,B5,B6}, {B1,B2,B5,B7}, {B1,B2,B5},{B1,B3,B5,B6}, {B1,B3,B5,B7}, {B1,B3,B5,B2}, {B1,B4,B5,B6},{B1,B4,B5,B7},{B1,B4,B5,B2} }

Neither {B1,B3,B5,B2 nor {B1,B4,B5,B2} are minimal cut sets since {B1,B2,B5} is a minimal cut set, hence these two cut sets are removed in Step 7, and we remain with:

CSs = { {B1,B2,B5,B6}, {B1,B2,B5,B7}, {B1,B2,B5},{B1,B3,B5,B6}, {B1,B3,B5,B7}, {B1,B4,B5,B6},{B1,B4,B5,B7} }

These are the minimal cut sets since there are no occurrences of an event and its complement in any of the cut sets, and there are no non-minimal cut sets left.

Proceeding to the “accident prevented” end consequence, we have:

In Step 1 we find CSs = { {B1,B2},{B1,B3},{B1,B4} }. In Step 3 we find for the dual fault tree CSsB = { {B5*},{B6*,B7*,B2*} }. By applying the & operator we get CSs = { {B1,B2,B5*}, {B1,B2,B6*,B7*,B2*},{B1,B3,B5*},{B1,B3,B6*,B7*,B2*},{B1,B4,B5*}, {B1,B4,B6*,B7*,B2*} }.

In Step 6 we see that {B1,B2,B6*,B7*,B2*} never will occur since B2 and B2* cannot occur at the same time, hence this cut set is removed, and we remain with:

CSs = { {B1,B2,B5*}, {B1,B3,B5*}, {B1,B3,B6*,B7*,B2*},{B1,B4,B5*}, {B1,B4,B6*,B7*,B2*} }

Note that the minimal cut sets only cover the situation where the initiating event occurs. Finding the dual fault tree to the left most fault tree could also find minimal cut sets for “nothing”, but this is usually not of interest, and hence omitted.

**Exercise 1:**

Use the above algorithm to verify the result in the PetriNet paper under “Documents for download” on It’s Learning