RAMS
Reliability, Availability,
Maintainability, and Safety

PK8200 - Risk influence modelling

Course compendium

Jørn Vatn

April 8, 2025 (to be updated)

Department of Mechanical and Industrial Engineering
Norwegian University of Science and Technology

Preface

This course compendium is developed for the course PK8200 - Risk influence modelling.

The reader is assumed to have background in basic probability theory and some skills in using ICT-tools. Many of the problems can be solved by using a dedicated Excel file with built-in functions which implement many of the formulas given in the compendium. It is also indicated how Python programming can be used.

Some of the problems can be solved without using ICT-tools, that is with an ordinary calculator. However, this will be tedious, and it is recommanded to practice with either Excel or Python.

Trondheim, April 8, 2025
Jørn Vatn

Contents

Chapter 1

Introduction

1.1 Background

blbl

Chapter 2

Introduction to Risk Influence Modelling

2.1 Introduction

This chapter gives an introduction to risk influence modelling and serves as an introduction to what we will cover in the course PK8200 – Risk influence modelling.

2.2 What is risk, and principle elements of a risk picture

A variety of risk definitions exist in the literature. In this presentation we only discuss risk definitions in relation to quantitative risk analysis. The baseline when risk is to be defined is to address threats to values of concern by the relevant stakeholders. This again forces us to be explicit regarding the *magnitude of the threat*, and the actual *consequences*, i.e., what values are at stake. It is common to link the definition of risk to *events* in one way or another. Often the term hazardous event is used to be explicit regarding the events that may result from the hazards or threats of concern. This then leads us to a conceptual risk definition saying that *risk is the uncertainty regarding the occurrence and severity of hazardous events*. To make the risk definition operational, we introduce three elements, events (e_i), probabilities (p_i) expressing the uncertainty regarding occurrence of the events, and finally the severity with respect to the values at stake ($\mathbf{S}_i$). This yields the quantitative expression of risk:

$$R = \{< e_i, p_i, \mathbf{S_i} >\} \tag{2.1}$$

When risk is expressed in terms of Equation (2.1) this is always done conditionally on a set of aspects which here are denoted the $\mathscr{D}$, $\mathscr{U}$ and $\mathscr{V}$. $\mathscr{D}$ represents the result of dialog processes and risk communication among stakeholders that elaborate on the values and preferences domain, such as who are exposed to threats, and whose needs in the society should be focused on. Further $\mathscr{U}$ represents the relevant information, the theories, the understanding, and the

assumptions which are the basis for the risk assessor, and finally $\mathcal{V}$ represents the result of any verification processes, e.g., third party verification. See Vatn (2012) for further discussions.

In the traditional or classical definition of risk, the probabilities in Equation (2.1) are interpreted as true properties of the system being analysed. Since we have limited data and information regarding the system it is impossible to reveal the exact values of these probabilities. It is then common to present uncertainty intervals for the risk measure. In the epistemic interpretation of risk it is the other way around. Then the basis is that there is *uncertainty* regarding whether the undesired events will occur (lack of knowledge), and the corresponding severity. Probabilities are used to express this uncertainty, and there is no additional uncertainty in the probability statements. However, as part of the documentation of the risk analysis uncertainty is qualitatively stated in terms of discussion of assumptions and simplifications. In relation to Equation (2.1) such arguments are stated as part of $\mathcal{U}$. Methods and models used in risk analysis are often not affected by the interpretation of risk in Equation (2.1). However, the way uncertainty is interpreted and presented will vary between the classical and the epistemic interpretations of risk.

To be explicit on how a risk picture would look like Figure 2.1 shows an example of such a risk picture related to one particular hazardous event (gas leakage). The uncertainty regarding the occurrence of the hazardous event is expressed by the probability figure $p = 0.1$, the severity is described by possible threats to the values at stake, i.e., possible fatalities. Here the number of fatalities is split into categories to simplify the presentation of uncertainty regarding the number of fatalities given that the hazardous event occurs. The frequencies, F_i, are the unconditional probability of each of the end consequences (fatality category).

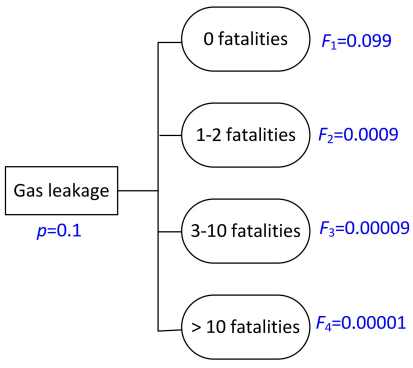


Figure 2.1: Example of risk picture related to one particular hazardous event

The following remarks are made in relation to Figure 2.1:

- Only one hazardous event is elaborated, meaning that we are filtering the risk picture
- Only number of fatalities is emphasized, leaving out e.g., injuries, environmental damage and loss of production (filtering)

- Conditions leading up to the hazardous event are not visualized, leaving out the entire causal analysis (hiding background information)
- Conditions affecting the probability distribution over the end consequences are not visualized (hiding background information)
- Location (x-y-z) and time of the event is not visualized (no zooming)
- No distinction is made regarding which gas leakage type is considered (aggregating over all types of gas leakage events)
- No distinction is made regarding which personnel groups are exposed (aggregating)
- Some consequences are merged into one category, i.e., more than 10 fatalities corresponds to the event that 11 persons are killed, the event that 12 persons are killed etc. All these events are merged into one event.

The above comments points towards various types of operators applied to the risk picture. Before we these operators the risk picture is now defined as:

Risk picture: A set of undesired events, the causes and factors that may contribute to the event, the possible consequences of the event with corresponding influencing factors, and uncertainties related to all these issues.

The operators applied on the risk picture are:

Filtering. With filtering we mean to filter out several aspects of the risk picture. Primarily filtering means to focus on only one hazardous event and/or a limited set of end consequences, e.g., only number of fatalities.

Aggregation. With aggregation we mean the process of summing more than one event, more than one cause etc. to give a sum of various events, causes and so on.

Merging. With merging we mean the process of grouping several similar outcomes into one category representing several outcomes.

Zooming. With zooming we mean to view part of the risk picture for a specific location (in space and/or time).

Hiding/unhiding. With hiding we mean to hide important information when presenting the complete risk picture. Typically we hide causes behind the hazardous event, factors that influence whether causes could lead to the hazardous event or not, and factors that influence the severity of the hazardous event, i.e., the probability distribution over the possible end consequences.

Due to limited resources when preparing the risk picture simplifications are made, merging is performed etc, hence it is not always possible to unmerge events, split up into a detailed set of failure causes etc.

2.3 Risk analysis

Risk analysis is the process of assessing the risk picture. Essentially this means to identify relevant hazardous events of concern, represent the corresponding values threatened in terms of a set of end consequences, and the uncertainty involved usually expressed by probabilities. Figure 1 shows one such hazardous event with related information.

In the following we discuss key elements in this process. The elements typically follow in chronological order, but going backwards is also often required in the analysis process.

1. *Identification.* Identification is two folded, first is the identification of hazardous events (or other undesired events) being the starting point of the analysis. The issue of identifying the *hazardous events* is not considered to be any principal problem. The second part of the identification process is much more demanding, i.e., to *identify relevant causes* to events, and factors and conditions affecting both the hazardous events, but also the severity given the event. In risk analysis it is equally important to identify conditions and factors that affect the situation in a positive way, as those having a negative impact.

2. *Structuring.* Structuring is an important step required before the modelling may start. Structuring means to present tacit knowledge, system understanding etc. in such a way that the risk analyst is able to start modelling. In order to cope with the problem of identifying all causes behind events, so-called complexity attributes (Vatn, 2012) should be identified and structured.

3. *Modelling.* In risk analysis there are two types of models, probabilistic and deterministic. A deterministic model is primarily used to describe relations between physical quantities and other real world observables. Examples of such models are fire and explosions models for calculating pressures given an ignition of a specific gas cloud. A probabilistic model is a model that *enables the risk analyst to apply the law of total probability* in an efficient way when expressing uncertainty, i.e., performing probability calculus. It is important to

emphasize that a probabilistic model is not a model of the world, but it is a model used to express uncertainty regarding observables in the real world. Examples of modelling tools are Markov analysis, fault tree analysis (FTA), event tree analysis (ETA) and Bayesian belief networks (BBN).

4. *Identification of the need of data.* All models will require input data such as failure rates, human error probabilities etc. Depending on the format and level of detail in the modelling, the main objective of this step is to be specific on the need for data and model parameters.

5. *Data collection and assessment of model parameters.* When the need for data is specified the next step is to collect data and in some cases also estimate/assign model parameter based on raw data and use of expert judgements.

6. *Run the model to establish the risk picture.* When the relevant models have been identified and built, they are feed with data. Then it is possible to run the models, and achieve the risk picture.

2.4 Building blocks for risk modelling

Mohaghegh et al. (2009) present a socio-technical risk analysis (SoTeRiA) framework which is a hybrid technique formalization, merging various classes of modelling techniques. In the following we briefly list the "building blocks" of SoTeRiA. For further elaboration we refer to Mohaghegh et al. (2009) and the references therein. The two last classes of techniques are extensions of the SoTeRiA framwork.

Formal probabilistic risk analysis techniques refer to methods that apply a logical construct to describe the system. They include classical probabilistic risk assessment techniques, such as Event Sequence Diagram (ESD), Event Tree (ET), Barrier Block Diagram (BBD) and Fault Tree (FT).

Regression-based techniques are common in economics and the social sciences. These techniques are used to distinguish true statistical causality from "spurious correlation". The process involves defining a set of variables and their relations, then "testing" all of the relations simultaneously. This is done by applying various techniques, such as Path Analysis or Structural Equation Modelling.

Bayesian belief nets (BBN) defines a methodology for representing causal connections that are "soft," "partial," or "uncertain" in nature. The applications of BBNs have grown enormously over

the past 30 years, with theoretical and computational development in many areas. A less strict formal realisation of such "soft" causal connections is modelling by use of risk influencing factors (RIF). A RIF is then a condition or factor that influences one or more parameters in e.g., a formal probabilistic risk analysis technique. BBN is often used to model interactions between RIFs. RIFs are discussed later in this document, and will be among the most central concepts in this course.

Process modelling techniques aims at modelling the primary production processes of the organization. At a first step semi-formal process technique are adapted and applied to represent the various processes (e.g., work process) in an organization. Then, for quantification purposes, it needs to be converted to a formal technique that is consistent with other techniques in the quantification framework.

Deterministic dynamic techniques are applied when there is enough information to establish "deterministic" relations among factors of the model or some parts of it. The deterministic modelling technique can be either analytical or simulation based. Examples of simulation-based techniques are Agent-Based Modelling (ABM) and System Dynamics (SD). SD shows significant capabilities for modelling certain human behaviour and decision-making processes, making it a good technique for modelling aspects of organizational behaviour. The strength of SD also lies in its ability to account for non-linearity in dynamics, feedback, and time delays.

Energy related impact techniques are used to model the situation after loss off control of energy sources representing a potential harm. These models include gas dispersion modelling, ignition modelling, fire and explosion modelling and structural integrity modelling. The models basically utilize deterministic physical and chemical laws, biological knowledge related to critical heat and pressure levels etc. Since initial and boundary conditions are not known in advanced, and many model parameters are uncertain, the deterministic models are supported by probabilistic approaches.

Recovery analysis techniques aim at modelling the recovering process after an emergency situation has occurred. It is reasonable to claim that the system now has become rather intractable which means that one single event tree model cannot capture the situation, see e.g., Hollnagel et al. (2006). Several scenarios need to be developed and there is a challenge to select a limited number of scenarios described at an appropriate level of detail to represent the recovery process. Also here the above listed techniques are relevant.

Note that in the literature a distinction is often made between systems which may be described

in terms of linear cause and effect relations and emergent systems with complex cause and effect relations. The latter is often denoted complex systems. The above classification does not explicit reflect differences between linear and complex cause and effects. A rough classification would be to say that the formal probabilistic risk analysis techniques apply for linear systems, whereas the remaining classes of techniques apply for emergent and complex systems. But rather than classifying the techniques, we believe that the challenge of approaching emergent systems is not to choose the appropriate technique, but rather to combine the class of techniques in a way that enable us to express our system knowledge in the most appropriate way.

Although the list of techniques provided above are quite impressing we do not claim to have any final approach to determine the true risk. The techniques are only tools we apply to express our uncertainty where the aim is to provide valuable decision support. This is also highlighted by e.g., Apostolakis (2004) where he discusses the value of risk analysis. He also points out limitations with current risk analysis practices, and points towards areas for improvement and further research. One such area is human performance during accident conditions. This points towards what is denoted *recovery analysis techniques* above. Challenges obvious occur when expressing human performance, but even more challenging would be to qualitatively express relevant accidental scenarios.

2.5 Risk modelling and risk influence modelling in a historical perspective

This section reviews important aspects of risk modelling required in order to present a risk picture. Vatn and Haugen (2012) distinguish between three types of risk analyses for use in the offshore oil and gas industry:

1. Strategic risk analyses are primarily aimed at developing a safe design and safe operating procedures. The objective is typically to assess a proposed design or an operation, to evaluate whether the risk level is acceptable and to identify potential risk reducing measures. They are characterized by being performed with a global perspective, in the sense that they are considering the effect on the risk level for a whole installation. These studies are primarily quantitative. They focus very much on technical aspects, with operational input primarily being limited to activity levels, such as the number of offshore supply vessels visiting the installation, the number of lifts being undertaken by cranes, the number of wells drilled etc. For the hydro carbon events the analyses usually starts with a blowout/gas leakage, and then proceed with event trees (ETA = Event Tree Analysis), fire and explosion models etc. to derive the risk picture. This means that they are not focusing on causes for the hazardous event as indicated in Figure 2.1. To some extent the barriers are mod-

elled by fault tree analysis (FTA). These type of analyses are often referred to as total risk analyses (TRA) or quantitative risk analyses (QRA).

2. Qualitative design analyses are more detailed and more specific than the strategic analyses, and they will typically have a system focus. The most prominent examples of such studies are HAZOP and FMEA. This could be performed e.g., on the mud system or on the Blowout Preventer (BOP). These are performed to verify the design in detail, to ensure that safe (and reliable) operation is possible. They usually have a strong technical focus and are typically the responsibility of the design team or onshore staff working with technical safety, similar to the strategic studies.

3. Operative risk analyses are different from the strategic studies in almost every respect:

   a) They are typically performed as qualitative studies, sometimes using a risk matrix to classify the identified hazards/events and to determine acceptability of the identified hazards.

   b) They are performed on a much more limited problem area, typically an operation that is being planned or is about to be performed or as support for a specific, limited decision. The analyses may often address major accident risk (although not necessarily), but the link to the global risk picture for the installation, as expressed through the strategic analyses, is usually weak in these studies.

   c) The responsibility for these studies may be different personnel groups, including onshore planning/operations groups or offshore personnel, responsible for performing the work.

As discussed by Vatn and Haugen (2012) the three types of analyses are usually separate analyses where they are seen as independent analyses not being able to support each other. Several attempts have been made in order to improve the quality of such analyses. An early attempt to make the strategic risk analyses more dynamic was the ORIM methodology, **?**. The idea behind this approach was to take existing QRA models and investigate these with respect to the most important parameters, e.g., a Birnbaum like measure, say $I^{\mathrm{B}}(i)$. Such a parameter could be the gas leakage frequency, or the safety integrity of a barrier. The next step was to investigate which of these parameters were most likely to change their value during the period between updates of the QRAs (typically every five year). The combination of *important parameters* and parameters that were *expected to vary* gave a list of parameters to include in a follow up regime.

Combining these gave raise to a methodology where the "critical" parameters were assumed to depend on a set of so-called risk influencing factors (RIFs). To monitor the RIFs, a set of risk indicators (RIs) were identified. For example for the RIF compentence, the number of certificates could be one RI, and the number of years in the department could be another RI. To

calculate the RIF a weighted sum of the RIs were used. Finally to get an updated risk profile, the critical parameter, say λ was updated according to

$$\lambda = \lambda_0 e^{\beta_0 + \beta_1 \mathrm{RIF}_1 + \beta_2 \mathrm{RIF}_2 + \ldots} \tag{2.2}$$

where λ_0 is the industry average value of the critical parameter, $\beta_0, \beta_1, \ldots$ are regression parameters and the RIFs are the numerical values of the risk influencing factors assessed by a weighted sum of risk indicators. The updated parameter, λ is then inserted in the existing QRA to get an updated risk picture reflecting the current state of the risk influencing factors.

To investigate the RIF-model in Equation (2.2), check the literature for Cox-proportional hazard models, e.g., Cox (1972).

To complete the approach a set of risk indicators (RI) were identified that were measuring the status of a RIF. The follow-up regime now was to obtain the values of the RIs every 3 months, calculate new RIFs based on the RIs, propagate this through the RIF model to get a change in the parameter. Now using the importance measure of the parameter, a new total risk picture could be established as well as the change in the risk level.

For various reasons the ORIM method was not implemented in the industry. One of the weaknesses in the ORIM was that it did not extend the existing QRA with respect to shed light on the failure causes. In the BORA project the focus was therefore changed to barriers and operative issues (BORA = Barrier and operative risk analysis), see Sklet et al. (2006). For critical conditions such as a gas leakage which in the QRA only is modelled by a single number found in generic databases, a detailed task analysis was conducted to reveal critical tasks that could lead to a gas leakage. For example review of real gas leakages revealed that many leakages were caused by maintenance of components and systems. This gave a much better understanding of what could go wrong, what where the critical tasks, and what RIFs influenced the probabilities in the model. This kind of modelling was very similar to HRA (Human Reliability Analysis) used in the nuclear industry. The BORA method is therefore seen as a significant improvement of the ORIM method since it have a much stronger link to the causes behind the initiating event (or failure of a critical barrier). A weakness of the BORA method was that it did not aim to update the QRA to see the total impact of the revised parameter (e.g., leakage frequency). The OMT project was a follow-up of the BORA project where the methodology was refined, see Gran et al. (2012) and Vinnem et al. (2012) for details. A weakness in the BORA model was that all the RIFs were threated in a flat hierarchy with only one level. It was recognised that there is a structure in the RIFs, where typically sharp-end RIFs are assumed to be influenced by blunt end RIFs (management issues). Further it was recognised that measuring the RIFs is demanding. Some companies are conducting internal audits to assess the current status of the RIFs on a regular basis (typically every second year). It is believed that this cannot be done without uncertainty, hence the RIFs were treated as stochastic variables (random quantities) and modelled

by Bayesian Belief Networks (BBN) taking a two level hierarchical structure into account. The OMT project also improved the interaction modelling between the RIFs, and proposed ways to treat common cause errors similar to methods used in HRA.

2.6 Risk influencing modelling – Principal content

A primary risk model such as a combination of fault- and event trees aims to capture the core element of the course of events in the various accident scenarios. Such models represent formal probabilistic risk analysis techniques. Roughly speaking this part of the model describes the linear cause and effects identified, structured by means of formal logical statements. The more "soft" relations cannot be expressed by formal logical structures such as AND and OR gates. For example the level of competence is assumed to influence the error probability of a critical task, but we cannot model this by e.g., a fault tree. Risk Influencing Factors (RIFs) and Performance Shaping Factors (PSFs) are often used to structure this part of the modelling. The various RIFs or PSFs are often structured by means of BBN techniques in order to model the influence on the basic events and barriers in the primary accident scenario model. In the following, we introduce important definitions used in risk influence modelling:

QRA parameter

Conditions that affect risk and included as single parameter in the quantitative risk model. Examples of QRA parameters are failure rates, on demand probabilities for safety systems and ignition probabilities. Primarily we assume that QRA models comprise formal probabilistic techniques like fault- and event trees, and energy related impact techniques, e.g., an ignition model.

2.6.1 Risk influencing factor (RIF)

A RIF is a factor or condition that influences the risk. There are three principal types of RIFs, (i) RIFs that are equal to a QRA parameter, (ii) RIFs where the value of a RIF is assumed to influence a QRA parameter, and where the influence is described in probabilistic terms, and (iii) RIFs that influence a parameter that is not explicitly modelled in the QRA. The situation in (ii) is the most important covered in this course. In most cases, the value of a RIF is not known, and the RIF is therefore treated as a random quantity.

2.6.2 Risk indicator (RI)

A quantity or condition that may be assessed or measured, where the value of the risk indicator is an indirect measure of a corresponding risk influencing factor. There are two principal types of

risk indicators, (i) resultant risk indicators which are reflecting the corresponding risk influencing factors, and (ii) controllable risk indicators, where it is possible by various efforts to set the value of the risk indicator, and hence change the value of the corresponding RIF. If the RIFs are treated as random quantities in the modelling, it is possible to derive the conditional probability distribution of a RIF given the value of one or more RIs. Among the models we consider, only the Risk_OMT treats RIFs as random quantities. In the Risk_OMT risk indicators are denoted scores.

2.6.3 Score (S)

A realisation of the true underlying value of a RIF. The term score were introduced in the Risk_OMT approach where the term score denote the summarized information regarding one RIF from interviews, surveys etc. A score is thus treated as a realisation (observation) of the true underlying RIF.

2.6.4 RIM parameter

A RIM parameter is a parameter in the risk influence model (RIM). There are several levels of RIM parameters, for example RIM parameters used in models to assign a QRA parameter given the value of a RIF, and RIM parameters used to link risk indicators (RI) or scores (S) to the RIFs.

2.7 Challenges in RIF modelling

In this section we discuss some of the main challenges with developing RIF models. A starting point for the discussion is that some quantitative risk analysis already exists. For example in the offshore oil and gas industry so-called total risk analyses (TRA) often also referred to as QRA exist for all installations. There are several reasons why we want to go into RIF modelling:

1. To get a more realistic risk picture taking “soft” factors into account
2. Link soft factors explicitly to the risk picture such that we can quantify the effect of risk reducing measures related to these soft factors
3. Establish a framework for updating the risk picture based on change of the value of risk indicators rather than updating the entire QRA to achieve a living risk picture.
4. By conducting the analysis gain more insight into risk influencing conditions, and hence be able to eliminate risk factors directly.

2.8 Identification of RIFs

To identify RIFs we may choose between a top down, or a bottom up approach. In a top down approach we start with an existing QRA and search for the most important RIF parameters. To quantify the importance of a RIF parameter in this context we take into account both (i) a technical importance measure like Birnbaums measure of importance (i.e., the change in total risk by a small change in the parameter value), and (ii) the likely change in the parameter. For example in the offshore oil and gas industry major accidents are mainly linked to well-control events, process events caused initiated by gas leakages, and structural damages caused by ship collisions. Thus the gas leakage frequency is a parameter with a high value according to the Birnbaums measure. If further, the gas leakage frequency is likely to change, or strongly affected by RIFs for which we have not really looked into, we have a good starting point to look for the most critical RIFs. In a bottom up approach we start with all possible RIFs we consider to have an impact on the total risk, and perform a screening without explicitly considering existing risk models.

Independent of a top down, or bottom up approach, we need to define the RIFs such that they relate to the existing risk models and QRA parameters. In some situations the RIF would be identical to the QRA parameter which is the easiest situation. In other situations, there will be a more or less direct link between the RIF and the QRA parameter. For example in human reliability analysis (HRA) the RIFs are denoted PSFs (performance shaping factors) and are linked directly to the so-called HEP (human error probability) which is a QRA parameter. The most challenging situation is when the level of details in the QRA is insufficient to really match the RIFs. For example in offshore oil and gas QRAs, the gas leakage frequency is modelled as one single number not taking the various failure causes into account. Failure causes for gas leakages spans over a range of technical, procedural and human error related issues, and if no model exist to map the failure cause level, it is also hard to link RIFs to an existing QRA parameter.

2.9 Extension of existing QRA models

As discussed above, existing QRA models often lack the level of details making them appropriate for linking RIFs to the QRA parameters. It is therefore often required to extend the existing QRA. Referring to the gas leakage example discussed above, a very simple approach is to split the gas leakage frequency into a set of failure causes followed by an assessment of the relative importance of each failure cause. The next step is then to link the various RIFs to one of the failure causes giving a good starting point for the modelling. Note that a cause here may be the failure of a safety barrier not directly linked or modelled in the QRA. In other situations it is required to develop new risk models to get confidence in the mapping of RIFs to the risk model.

For example in the Risk_OMT methodology several task analyses were carried out to really catch critical activities during maintenance that influences the gas leakage frequency. An advantages developing detailed models is that we also get a better qualitative understanding of those issues that may cause e.g., a gas leakage. Such understanding is of great importance when searching for explicit risk reducing measures.

2.10 Defining the scale of the RIFs

Every RIF in a risk influence model has a value (known or unknown) which depends on the scale being used. Now, let r be the value of a RIF. A neutral scaling regime would be to define $r = 0$ to be the industry average of a RIF, $r = -1$ be the worst case we can imagine, and $r = +1$ be the best we can imagine within a reasonable time horizon. Another approach is to use an arbitrary scale, for example in the ORIM model $r = 1$ corresponds to the worst case, and $r = 5$ to the best case. In the BORA and Risk_OMT models character values were used, where $r = A$ corresponds to best practice, and $r = F$ corresponds to the worst case, or unacceptable state. It is recommended to use the same scale for all RIFs. Later on we will also discuss risk indicators (or scores) as a means to assess the value of a RIF. It is recommended to use a matching scale for risk indicators/scores. However, this is more demanding, since for example a risk indicator may be measured in terms of e.g., n = number of personnel having a certain formal certificate. In such cases, a mapping is required, for example $n = 0$ corresponds to the character F, $n \in [1,2]$ corresponds to E and so on. Such a mapping is not straight forward and requires careful considerations.

2.11 Structuring RIFs

Usually more than one RIF are influencing a QRA parameter. In simple models a weighted sum is calculated to represent all the RIFs, and this sum is then used to adjust the QRA parameter. In more advanced modelling, two aspects are considered. The first aspect relates to the fact that a weighted sum will not take into account interaction effects between the RIFs. In some situations a bad value of two or more RIFs is considered more critical than the individual contribution from these two bad values, i.e., there are some interaction effects we would take into account. The second aspect relates to dependencies between RIFs. For example if we split into “sharp end” RIFs (e.g., time pressure) and “blunt end” RIFs (e.g., management of work organization) the latter RIF is assumed to influence the first RIF. If we collect data and combine into scores to reflect the value of the RIFs on different levels, we need to develop an influence model to connect the RIFs. To accomplish this BBN methods are considered to be a good approach.

2.12 Linking risk indicators and scores to the RIFs

In simple models we link risk indicators to the RIFs by simple weighting formulas. If RIFs are treated as random quantities, the risk indicators (or scores) are only considered to be *indicators* for the true underlying value of the RIF. We then need to express how strong evidence a value of a risk indicator really is. A simple way to express this is to say that given a value r of a RIF, the risk indicator or score, will be a random variable, say S, where $\mathrm{E}[S] = r$, and in addition a precision parameter is required to express $\mathrm{Var}(S)$. This will enable inference, i.e., assessing a probability distribution over the RIF, or simultaneous distribution over a set of RIFs by e.g., BBN methods.

2.13 Linking RIFs to the QRA parameters

Independent of whether RIFs are treated as random quantities, or fixed known values, it is necessary to link the various RIF values to corresponding QRA parameters. If r is the value of a RIF, and p is a corresponding QRA parameter, we need to establish a functional relation:

$$p = f(r) \tag{2.3}$$

In order to establish such a relationship we often ask what will be the value of p when the RIF take the best and the worst value respectively. For values of the RIF in-between we often choose between linear or geometric interpolation.

Let p_{L} and p_{H} be the lowest and highest value the QRA parameter of interest can take respectively. Further let r_{L} and r_{H} be the lowest and highest value a RIF value can take respectively[1] (considering only one RIF). A linear interpolation is now given by:

$$p(r) = p_{\mathrm{L}} + \frac{(r - r_{\mathrm{L}})(p_{\mathrm{H}} - p_{\mathrm{L}})}{r_{\mathrm{H}} - r_{\mathrm{L}}} \tag{2.4}$$

and similarly a geometric interpolation is given by:

$$p(r) = p_{\mathrm{L}} \left(\frac{p_{\mathrm{H}}}{p_{\mathrm{L}}} \right)^{\left(\frac{r - r_{\mathrm{L}}}{r_{\mathrm{H}} - r_{\mathrm{L}}} \right)} \tag{2.5}$$

Generally a geometric interpolation is recommended if the range of the parameter variation spans more than one decade.

If we have more than one RIF that is influencing a QRA parameter, we need to develop an interaction model to combine the RIFs.

[1] We here assume that a low value of the RIFs gives a low value of the corresponding QRA parameter to simplify the presentation

2.14 Structuring QRA parameters, RIFs and RIs/scores

Ideally we would like to make a clear distinction between QRA parameters, RIFs and RIs. The QRA parameter represent the quantity used in the QRA models which are the starting point of the risk influence modelling. Then RIFs are introduced to represent conditions that are influencing the QRA parameter, where the aim is to give a RIF a theoretical interpretation like competence of maintenance personnel. In order to assess the value of a RIF, the RIs are introduced. In some situations we build a very rigid risk influencing model by formal e.g., a formal BBN model. For example in the Risk_OMT model some 5-8 RIFs were introduced in a hierarchy to show the influence on a particular QRA parameter. In the Risk_OMT the RIFs were more or less identical to variables used in safety audits, and hence there were a one to one relation between the RIF and the RI (where the term score was used rather than a risk indicator). In other situations less effort is made to structure the various RIFs, and at the extreme we may leave out the explicit definition of RIFs and link the risk indicators directly to the QRA parameter, for example by a weighted sum.

2.15 Hybrid vs full BBN models for the entire QRA modelling

If the RIFs are treated as random quantities, Equation (2.3) may be used to get a probability distribution over the parameter p. This represents a so-called parameter uncertainty which we in principle may propagate in the QRA model. Since fault- and event trees in principle may be converted to BBNs, and since a BBN is also used to model the distribution over various r values, these BBN models may be combined to give a full BBN model for the total quantitative risk model. Experience from the Risk_OMT project has shown that this is impossible due to memory and time constraints within existing BBN implementations. To create an approximated full BBN model requires careful consideration, and deep knowledge into application of BBN modelling. A hybrid model applies to Equation (2.3), but rather than finding a probability distribution over p, the uncertainty regarding the RIFs is being integrated to give an expected value of the QRA parameter. This value could then be used in existing QRA models represented by fault and event tree rather straight forward. Such an approach, is however, not conservative since if the same RIF influences several QRA parameters, we then will ignore the “state of knowledge” dependency in the value of the RIFs, and hence “underestimate” the risk. More research is required, to find ways to improve such hybrid modelling.

Chapter 3

Risk_OMT – Hybrid approach

3.1 Introduction

This chapter describes the technical aspects of the hybrid implementation of the Risk_OMT model. The Risk_OMT offers two modelling approaches. The full-BBN (Bayesian Belief Network) approach utilize BBN modelling both for the soft influences between risk influencing factors (RIFs) and the formal probabilistic relations described by fault- and event trees. The hybrid implementation of the Risk_OMT model uses a BBN specification of the relation between the RIFs but uses ordinary processing of the fault- and event trees. See Appendixes B, C and D for technical details if you are not familiar with such modelling.

In the fault and event trees failure of an activity is divided into failures of *omission* and failure of *execution*. Failure of omission denotes whether or not the prescribed activity is carried out. Failure of execution denotes inadequate actions that may cause failures, e.g., acts performed in a wrong sequence, at wrong time, without required precision etc. Failure of execution is seen as results of *human errors* and *violations*. Human error is further divided into *mistakes* and *slips & lapses*, where mistakes involve actions that are based on failure of interpretation of procedures, and/or failures of judgemental/inferential processes involved in the prescribed activity. This category does not distinguish between whether or not the actions directed by this judgement activities run according to the actors plan. Typical mistakes are inadequate judgement/conclusion due to intrinsic conditions such as competence, fatigue, mode etc., and extrinsic conditions such as communication, information, work load, time pressure etc. Slips & lapses involve actions that represent unintended deviation from those practices represented in the formal procedures. This is deviation due to error in execution and/or the storage stage of an action sequence. For our purpose, this category represents only actions where there is no intended violation, failure of interpretation of procedures and judgement failures prior to the action carried out. In the Risk_OMT model separate BBNs are developed for the RIF structure for (i) mistakes, (ii) slips & lapses, and (iii) violations. For failures of omission currently no RIF model is derived.

3.2 Risk influencing factors

A Risk Influencing Factor (RIF) represents a condition or a situation that influences the risk in a risk model. In this presentation we always assume that the RIFs are influencing the risk through parameters used in the risk model. In the Risk_OMT we mainly focus on different organisational conditions that have a theoretical and/or empirical grounded influence on the possible deviations from required actions, and hence should be reflected in probability assignment of errors or failures. Further, Risk_OMT operates with 2 levels of RIFs which links the organisational conditions (RIF Level 1) to strategic management decisions (RIF Level 2). In the current Risk_OMT implementation it is only RIFs on level 1 that directly influence the basic event probabilities.

Note that the term risk is interpreted differently within the society of risk analysis. In a classical risk analysis framework risk is seen as a property of the system being analysed. Further the probabilities in a risk model are considered to represent some true likelihood of e.g., component failures and human errors. With such an interpretation we may think of the RIFs as a way to establish a true causal link between some conditions and the basic event probabilities. In an epistemic interpretation of risk the main focus is on uncertainty. Risk is essential uncertainty regarding the occurrence and severity of undesired events. In such a framework basic event probabilities are not considered as some true values, but are expressions of our uncertainty regarding the occurrence of the basic events. The RIFs will then represent conditions that we take into account when assigning probabilities (expressing uncertainty) to the basic events, but we do not consider any causal link as for the classical interpretation.

The Risk_OMT modelling framework is an extension of the BORA release model (Aven et.al. 2006). There are two major changes in the Risk_OMT model compared to the BORA release model. Whereas the BORA release model combined the RIFs on the same level, the Risk_OMT model introduces a hierarchy between the RIFs. Further the BORA release model considered the RIFs to be known without any uncertainty. In the Risk_OMT model RIFs are still considered to be theoretical constructs that influences the risk, but we do not have exact knowledge regarding the value of the RIFs, and hence they are treated as stochastic variables (random quantities).

Formally we use the term score to denote the summarized information regarding the RIFs form interviews, surveys etc. A score is thus treated as a realization (observation) of the true underlying RIF. In the BBN this corresponds to an arrow from the RIF to the corresponding score. The scoring system is based on characters A to F, where A corresponds to best industry practice, and F corresponds to an unacceptable state with respect to the actual RIF.

Figure 3.1 shows an example of a RIF structure. Level 1 RIFs point to the basic events in the fault tree showing that level 1 RIFs influences the basic events. Level 2 RIFs influence the level 1 RIFs, and there is an arrow from the RIFs to the scores to indicate that the scores are treated as realizations (observations) of the true underlying RIFs.

Examples of level 1 RIFs are *technical documentation* and *time pressure*. Corresponding level

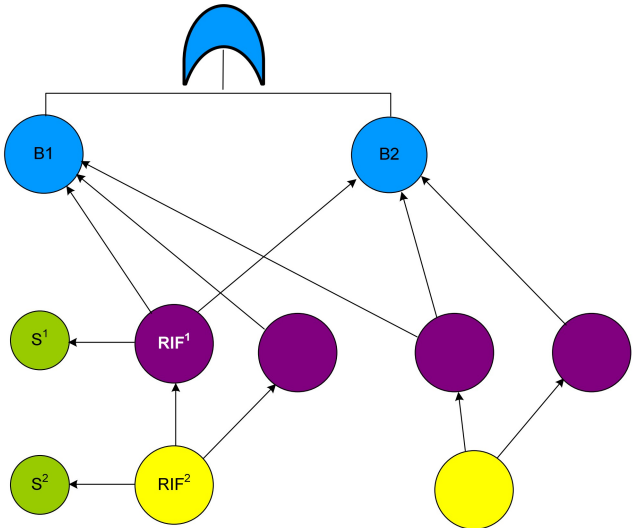


Figure 3.1: RIFs on two levels with scores and relation to basic events in a fault tree

2 RIFs are *management of information* and *management of tasks* respectively.

3.3 Impact of the level one RIFs on the basic events

In the hybrid Risk_OMT model the impact of the RIFs are explicitly modelled via the probability of occurrence of a basic event or a barrier in the fault or event tree. We now consider basic event number i. Three quantities span the sample space for the basic event probability for this event:

$q_{i,A}$ = average basic event probability corresponding to average industry practice, i.e., all RIFs equal to the character C.

$q_{i,L}$ = lowest basic event probability corresponding to the best practice in the industry, i.e. all RIFs equal to the character A.

$q_{i,H}$ = highest basic event probability corresponding to the an unacceptable industry practice, i.e. all RIFs equal to the character F. It is not expected to observe RIFs of character F.

Often $q_{i,L}$, and $q_{i,H}$ are specified indirectly by error factors, say $\mathrm{EF}_{i,L} = q_{i,A}/q_{i,L}$ and $\mathrm{EF}_{i,H} = q_{i,H}/q_{i,A}$ respectively. In the modelling it will be convenient to represent the RIFs by numeric values rather than character values. It is convenient to map the character values on the interval [0,1]. Splitting this interval into 6 sub intervals, and mapping the character value into the centre gives the value 1/12 for an A, the value 1/6+1/12 = 3/12 for a B , the value 2/6 + 1/12 = 5/12 for a C up to 11/12 for a value F. etc. In the following we will always use this mapping in the numeric quantifications. In some situations we may use a more differentiated labelling than

the pure characters, i.e., we may succeed the characters with extra plusses (+) or minuses (-). For example we may use that A++ corresponds to a value 0, A+ corresponds to a value 1/24 etc.

If we use r as a value of a weighted sum of the RIFs influencing the basic event probability we now introduce $q_i(r)$ to describe the functional relationship between the RIF value (r) and the basic event probability. We have that $q_i(0) = q_{i,L}, q_i(5/12) = q_{i,A}$ and $q_i(1) = q_{i,H}$. In between these values we may either use linear or geometric interpolation. For high variance it is recommended to use a geometric interpolation.

We will now assume that there are totally J RIFs that are influencing basic event i. Let $\mathbf{R} = [R_1, R_2, \ldots, R_J]$ be a vector of stochastic variables to represent these (standardized) RIFs, and let $p_{\mathbf{R}}(\mathbf{r}) = \Pr(R_1 = r_1, R_2 = r_2, \ldots, R_J = r_J)$ be the joint probability distribution over these RIFs.

If the value of the RIFs are given, we assume there is a relation between the basic event probability and the values of the RIFs:

$$q = q(\mathbf{r}) \tag{3.1}$$

Note the notation here, the upper case R is used for the RIF as a random quantity, and the lowercase r is used for a particular value.

Each RIF might have different weight with respect to the influence on the basic event probability. Let w_j be normalized weight for RIF j. A first approximation for the total impact of the RIFs on the basic event probability is given by:

$$q_i = \sum_{\mathbf{r}} q_{\mathrm{L}} \left(\frac{q_{\mathrm{H}}}{q_{\mathrm{L}}} \right)^{w_1 r_1 + w_2 r_2 + \ldots} p_{\mathbf{R}}(\mathbf{r}) \tag{3.2}$$

where $\sum_{\mathbf{r}}$ represents the sum over all possible values of $\mathbf{R}$. Equation (3.2) is then used to establish the basic event probabilities to use in the fault and event tree part of the hybrid risk analysis. It is rather complicated to establsih $p_{\mathbf{R}}(\mathbf{r})$, so it is not straight forward to apply Equation (3.2).

3.4 The beta distribution to describe uncertainty regarding the RIFs

A mathematical convenient probability distribution to use for continuous variables on the interval [0,1] is the beta distribution. Although the scoring of the RIFs are on an ordinal level, a continuous ratio scale seems appropriate for the modelling. The probability density function of the beta distribution is given by:

$$f(r) = r^{\alpha-1}(1-r)^{\beta-1}/\mathrm{B}(\alpha, \beta) \tag{3.3}$$

where $B(\alpha,\beta) = \Gamma(\alpha)\Gamma(\beta)/\Gamma(\alpha\beta)$ is the beta function, and $\Gamma()$ is the gamma function. α and β are parameters in the distribution.

If R is beta distributed with parameters α and β the expected value and variance are given by:

$$\mathrm{E}[R] = \frac{\alpha}{\alpha+\beta} \tag{3.4}$$

$$\mathrm{Var}(R) = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)} \tag{3.5}$$

Note that the beta distribution represents a conjugate prior distribution for the binomial distribution. Thus if the beta distribution is used to describe the parameter r in a binomial distribution with prior parameters α_0 and β_0, then the posterior distribution is also beta distributed with parameters $\alpha_0 + x$ and $\beta_0 + n - x$ where x is the number of successes and n is the number of trials of an experiment provided to update the prior distribution, i.e., the posterior distribution is a beta distribution with parameters $\alpha = \alpha_0 + x$ and $\beta = \beta_0 + n - x$.

3.5 Updating the RIF distributions based on the scores

The above results in Equations (3.4) and (3.5) do not apply directly to our situation since we will not get observations from a binomial trial but rather one observation considered to be a realisation of the true RIF.

Let α_0 and β_0 be the parameters in the prior distribution of the RIF prior to observing the score S. Given the true value of the RIF, say r, it is reasonable to assume that $\mathrm{E}[S|r] = r$, and further we assume that it is possible to specify $\mathrm{Var}(S|r) = V_S$. We now make the following argument: We will use the value of the score, say s, by translating the information to a binomial situation, e.g., finding x and n. This is done due to the simple result that exists for the binomial situation. Since X/n is an estimator for r in the binomial situation, and the score S is the estimator for r in our situation, it seems reasonable to require:

$$\mathrm{Var}(X/n) = r(1-r)/n = \mathrm{Var}(S) = V_S \tag{3.6}$$

Thus, if we know V_S and replace r with it's estimate s, we should have:

$$n = s(1-s)/Var(S) = s(1-s)/V_S \tag{3.7}$$

further since x/n and s both are estimates of r, we set $x = s \cdot n$. Utilizing the result from the binomial situation where the posterior distribution is beta distributed with parameters $\alpha_0 + x$ and $\beta_0 + n - x$ we will in our situation approximate the posterior distribution with a beta distribution

with parameters:

$$\alpha = \alpha_0 + s^2(1-s)/V_S \tag{3.8}$$

$$\beta = \beta_0 + s(1-s)/V_S - s^2(1-s)/V_S = \beta_0 + s(1-s)^2/V_S \tag{3.9}$$

□

Exercise 3.1

Find the expected value and the variance of the posterior distribution with the parameters obtained by Equations (3.8) and (3.9). Compare this result with the expected value and variance of the weighted sum of the prior mean and the score where the reciprocal variances are used as weights. □

3.6 Level 2 RIFs

For level 2 RIFs it is straight forward to use the result in Equations (3.8) and (3.9) to find posterior distributions for the RIFs. Various principles may be used for specifying the prior distribution. In order to have a method that is data driven as far as possible, it seems reasonable to apply $\alpha_0 = \beta_0 = 0.5$ corresponding to Jeffreys prior (Jeffreys, 1946).

Exercise 3.2

Apply Jeffreys prior together with Equations (3.8) and (3.9) in order to find the expected value and the variance of the posterior distribution for scores corresponding to the characters A, B, ..., F. Present the result in a table for V_S equal to 0.2^2, 0.1^2 and 0.05^2 respectively.

3.7 Level 1 RIFs

We will start by assigning the posterior distribution of level 1 RIFs given the value of the parent RIF, i.e., the corresponding level 2 RIF denoted P (i.e., parent). Given the value of the level 2 RIF, say $P = p$, it is reasonable to specify a prior distribution of the RIF, say R, with expected value $\mathrm{E}\left[R|P = p\right] = p$. Thus the structural dependencies between the parent RIF and the child RIF is considered to give the same expectation. But how strong is the structural dependency, i.e., the influence of the parent RIF on the child RIF? Such a structural dependency may be expressed by the variance of the child RIF, i.e., $\mathrm{Var}(R|P = p)$. To make the model simple we assume that $\mathrm{Var}(R|P = p) = Var(R) = V_P$ where it is possible to specify V_P independent of the actual value of the parent. V_P may be considered as a measure of the structural dependence in the model. Proposed values for the structural dependency are $V_P = 0.2^2$ (low dependency), $V_P = 0.1^2$ (medium

dependency) and $V_P = 0.05^2$ (high dependency). Prior to observing the score, it seems reasonable to express the prior distribution of the RIF with a beta distribution with expected value p and variance V_P.

Exercise 3.3
Use Equations (3.4) and (3.5) to show that we may obtain the prior parameters in this situation by:

$$\beta_0 = \left(\frac{p(1-p)}{V_p} - 1\right)(1-p) \tag{3.10}$$

$$\alpha_0 = \frac{p\beta_0}{1-p} \tag{3.11}$$

□

Conditional on the value of the parent level 2 RIF; i.e., $P = p$, and the structural influence between these RIFs, the prior distribution may be obtained by applying the parameters in Equations (3.10) and (3.11). Given the score $S = s$ of the level 1 RIF we apply (3.8) and (3.9) to find the conditional posterior distribution, i.e., given the parent value. In order to find the unconditional posterior distribution, we may integrate over the posterior distribution of the parent node. It is, however, important to stress that we do not need the unconditional posterior distribution of the child RIFs, i.e., the level 1 RIFs. In Equation (3.2) we need the joint distribution over the level 1 RIFs that directly influences the basic event probability. From the theory of BBN, we know that the level 1 RIFs are independent *given* their parents, i.e., the level 2 RIFs. This means that we may multiply the conditional posterior distributions for level 1 RIFs to find the required $p(\mathbf{r})$ in Equation (3.2) and then integrate over the joint posterior distribution of the level 2 RIFs.

In Equation (3.2) we did assume that the RIFs were made discrete, i.e., each RIF takes a finite number of values. This is done in order to simplify calculations. Since the scores are measured on six different values, it seems reasonable to use 6 values for each RIF both on level 1 and level 2. Then we may for the posterior distribution of the level 2 RIFs calculate a point probability for each interval, i.e., [0,1/6], [1/6,2/6] etc. Similarly, given the (discrete) values of the level 2 RIFs, we may calculate point probabilities for the level 1 RIFs, and $p(\mathbf{r})$ is then found by applying the law of total probability.

3.8 Finding basic event probabilities based on the RIF structure

Up to now we have obtained the following:

1. For the second level RIF's:

- We use Jeffreys prior before we have access to the scores
- When we get the scores, we update the prior to the posterior by applying Equations (3.8) and (3.9)
- The posterior is a Beta-distribution, currently not discretized.

2. For the first level RIF's:
   - Given the value of the parent, say $P = p$, the prior distribution parameters are given by Equations (3.10) and (3.11)
   - When we get the scores, we update the prior to the posterior by applying Equations (3.8) and (3.9)
   - The prior is only known for a given value of the parent, i.e., if we know $P = p$. However, we do not know the value of the parent, i.e., second level RIF.
   - The posterior is a Beta-distribution, currently not discretized.
3. Equation (3.2) may be used to find the basic event probabilities, but it is not easy to obtain the joint probability distribution over all RIF's.

3.8.1 Discretization

Assume that a stochastic variable R is Beta-distributed with parameters α and β, and that the cumulative distribution function, $F_R(r)$ is known. That is, we have a numerical routine to calculate $F_R(r)$. To discretize the distribution into K values, we use the midpoint for each value, i.e., $p_k = k/K - 1/(2K), k = 1,\ldots,K$. In our case $K = 6$ and the midpoints corresponds to the character scores $A, B, \ldots, F$.

To find the point probabilities we use:

$$\Pr(R = p_k) = F_R(k/K) - F_R((k-1)/K) \tag{3.12}$$

Exercise 3.4
Write a simple code (VisualBbasic, Matlab, Python, Fortan or C) to find the point probabilities for each interval [0,1/6], [1/6,2/6], .., given the parameters in the posterior distribution. □

3.8.2 Level 2 RIFs

Assume we have J second level RIFs, and let

$$\pi_j(k) = \Pr(R_j^2 = p_k) \tag{3.13}$$

for the jth second order RIF, i.e., R_j^2. To obtain $\pi_j(k)$ we use Equation (3.12) with the posterior distribution for the second order RIFs. Thus we have established a way to find the probability of each $p-values$ used in the conditional posterior distribution for the first level RIFs.

3.8.3 Level 1 RIFs

The basis for the conditional prior distribution for the first level RIFs was to include the structural dependency, V_P, and a given value of the parent, say p_k to find the conditional prior distribution parameters α_0 and β_0. When the value of the score $S = s$ is obtain the conditional posterior distribution can be obtained. As for second level RIFs we can also use Equation (3.12) to discretize the level 1 RIF posterior distributions.

Exercise 3.5
Consider a situation with one level 2 RIF, two level 1 RIFs influenced by the level 2 RIF. Write a simple code to find the unconditional distribution over the weighted sum of the level 1 RIFs. Make the code flexible such it is possible to specify the weights, the scores, the structural influences V_P's and the variances of the scores V_S's. Hint: First find the expected value (μ) and the standard devation (σ) of the weighted sum. Then fit a beta-distribution with α and β matching μ and σ. Then, discretize to find point probabilities for A,B,...,F. □

Exercise 3.6
Discuss extension of the model in the previous exercise where there are more than two level 1 RIFs for each level 2 RIF, and where there are more than one level 2 RIF. Hint: Since each level 1 RIF is influenced by one and only one level 2 RIF, the subset of level 1 RIFs with common parent level 2 RIF may be treated separately. Discuss why this will reduce the number of combinations to run through. Also discuss how to implement the solution if this savings should be obtained. □

3.8.4 Compiling the results for level 2 and level 1 RIFs

In our case, the value of the parent RIF, p_k, is not known, we only have discretized posterior distributions for the second level RIFs. We also recognize from Bayesian Belief Network theory that first level RIFs are independent only if the second order RIFs are know. This is important when we shall obtain $p_{\mathbf{R}}(\mathbf{r})$ to be used in Equation (3.2).

Actually, in the first place we only need a subset of the RIFs, i.e., $\mathbf{R}^1$ = the first level RIFs. If we fix all level 2 RIFs, i.e., assign a value to R_j^2 all children (level 1 RIFs) of level 2 RIF j are independent. Hence, for we can multiply the marginal posterior distributions for the level one RIFs to obtain $p_{\mathbf{R}}^1(\mathbf{r}^1)$.

To simplify notation assume that there are I_j level one RIFs that are children of level two RIF j. If we only consider these in the joint distribution, $p_{\mathbf{R}}(\mathbf{r})$, we can write the joint distribution as a product of the marginal distributions if the corresponding parent RIF (level two) is known, say takes the value p_k. If the first level RIFs are discretized similarly as for the second order RIFs, the conditional contribution to q is given by:

$$q_{j,k} = \sum_{\mathbf{r}} q_{\mathrm{L}} \left(\frac{q_{\mathrm{H}}}{q_{\mathrm{L}}} \right)^{\sum_{i=1}^{I_j} w_i p_l^i} \prod_{i=1}^{I_j} \rho_i(l,k) \tag{3.14}$$

where $\rho_i(l,k) = \Pr(R_i^1 = p_l | R_j^2 = p_k), l = 1,\ldots,6$ and where the sum $\sum_{\mathbf{r}}$ is over all possible combination of the first level RIF values, i.e., in total $\left(I_j\right)^6$ combinations. Note that $\rho_i(l)$ is found by integrating the conditional posterior PDF over the lth interval using Equation (3.12).

p_l^i is used to denote that the i'th RIF takes the value p_l, e.g., $p_2^1 = 3/12$ if the first level one RIF is an B.

To calculate $q_{j,k}$ from equation (3.14) we need to run through all the $\left(I_j\right)^6$ combinations of the $\mathbf{r}$-vector. Assume we have written a function `nxtComb(r)` that gives the next combination of the vector `r`, and returns `TRUE` as long as there are more combinations, we may use something like:[1]

```
:
q_jk=0
r=[0,1,1,....,1] ' Initial vector
Do While nxtComb(r)
   q_jk = q_jk + new contribution from current r-vector
Loop
:
```

to calculate $q_{j,k}$.

Exercise 3.7
Assume that you have n RIFs where each RIF may take m different values. Propose an algorithm to generate all possible combinations of the n RIFs, i.e., to implement the `nxtComb()` function. Use the `nxtComb()` function to implement the solution to Exercise 3.5. Then, change to three level 1 RIFs. □

In Exercise 3.7 we calculated the mean value of the weighted sum of the RIFs and the point distribution reflecting the uncertainty in the weighted sum of the RIFs. However, the final use of

[1] Note that the code assumes discretization uses indexes running from 1 to 6. In Python this should be from 0 to 5. Also note that the first element in the r-vector is 0, such that the first call to NextComb gives [1,1,...,1]. Similar for Phtyon where the first call should give [0,0,...,0].

the model is to propoagte the uncertainty in the weighted sum of the RIFs into the human error probabiltiy, i.e., HEP = $q = q_{\mathrm{L}}\left(\frac{q_{\mathrm{H}}}{q_{\mathrm{L}}}\right)^{w_1R_1+w_2R_2+\ldots}$. To obtain the HEP the easisest way is to use the mean value of the weighted sum from Exercise 3.7 and plug it in in stead of the weighted sum, i.e., $w_1R_1 + w_2R_2 + \ldots$. This will obviously ignore all uncertainty in the RIFs. A better approch would be to use the discretized distribution, i.e., the

$$q = \sum_{i\in\{A,B,\ldots,F\}} q_{\mathrm{L}}\left(\frac{q_{\mathrm{H}}}{q_{\mathrm{L}}}\right)^{i} p_i$$

where $\{A, B, \ldots, F\}$ are the numerical values, i.e. $A = 1/6, B = 3/12, \ldots$ and p_i are the corresponding point probabilities obtained by Exercise 3.7.

The ultimate approach is to modify the code and calculate $q_{\mathrm{L}}\left(\frac{q_{\mathrm{H}}}{q_{\mathrm{L}}}\right)^{w_1r_1+w_2r_2+\ldots}$ for the various RIF-value combinations and multiply with the probability for each combinations, and finally add the contributions.

Compare the results for each of these three approaches. Use $q_{\mathrm{L}} = 0.0001$ and $q_{\mathrm{H}} = 0.05$. □

Equation (3.14) represents the "contribution" from RIF number j given it takes the value p_k. Since $\pi_j(k)$ is the probability that the second level RIF number j takes the value p_k the law of total probability may be used to find the *unconditional contribution* to q:

$$q_j = \sum_{k=1}^{6} q_{j,k}\pi_j(k) \tag{3.15}$$

Finally, taking all J level two RIFs into account:

$$q = \left(q_{\mathrm{L}}\right)^{-J+1} \prod_{j=1}^{J} q_j \tag{3.16}$$

Exercise 3.8
Comment on why the term $\left(q_{\mathrm{L}}\right)^{-J+1}$ is introduced in Equation (3.16).

3.9 Interactions between RIFs

Before we start discussing interaction, one should reflect on differences between "interaction (synergy)", "common cause" and "cascading effects".

So far we have assumed that the influence of one RIF on the basic event probabilities is independent of the value of the other RIFs in the basic Risk_OMT model. In many situations it might be reasonable to believe that for example the negative influence of a very bad RIF is higher if one or more of the other RIFs also have a very bad value compared to more moderate

values of these RIFs. Such effects are denoted interaction effects.

In the modelling of interaction effects we introduce sub sets of the total set of RIFs to represent a potential for interaction. In principle we may have several such sub sets. In the presentation we assume that we only consider one subset. Let $\mathscr{I}$ be a sub set of level one RIFs where we will consider interaction effects.

There could be arguments supporting that interaction only comes into play when two or more RIFs are in a very bad condition. We therefore assume that (negative) interaction only applies if the RIFs are worse than the average value, i.e., a C.

In order to simplify the modelling of interaction effects we assign a weight, $w_{\mathscr{I}}$ of the interaction effect which is relative to the weight of the various RIFs in the interaction sub set, $\mathscr{I}$. For each RIF in the sub set $\mathscr{I}$ we then may find a total weight of the RIF in addition the original weight of the RIF, i.e.,

$$w_{\mathscr{I},i} = w_i w_{\mathscr{I}} f \tag{3.17}$$

where w_i is the original weight of the RIF, and f is a correction factor. If one or more of the RIFs have a value better than the average RIF-value (C) we set $f = 0$.

If all the RIF values in $\mathscr{I}$ have the worst value (F), we set $f = 1$. For values between we apply a linear transformation:

$$f = \frac{\sum_{i \in \mathscr{I}} (r_i - C)}{\sum_{i \in \mathscr{I}} (F - C)} \tag{3.18}$$

where $C = 5/12$ and $F = 11/12$.

It is then easy to verify that if all RIFs take the value "C" we get $f = 0$, and if all RIFs take the value "F" we get $f = 1$ corresponding to our assumptions. The total impact of the RIFs on the basic event probability is now:

$$r = \sum_i w_i r_i + \sum_{i \in \mathscr{I}} w_{\mathscr{I},i} r_i \tag{3.19}$$

where we have summed the interaction effects for one sub set of interactions.

3.10 Common cause failures

Note the difference between interaction between RIFs and common cause failures. In Risk_OMT interaction means that the negative impact of one bad RIF is extra strong if also other RIFs are bad. The interaction is only expressing the impact of bad RIFs, nothing is said regarding the likelihood of bad values.

Common cause failures relate to dependencies, i.e., the likelihood of one failure depend on

whether another event has occurred. In our context this also means that interaction effects are used for the RIFs, whereas common cause failures are used for the basic events.

There are many conceptual frameworks for common cause failures. Here we will stick to the β factor model. In the β factor model we assume that a *portion* of the failure rate or failure probability represent common cause failures, and will cause two or more basic events to fail simultaneously. This portion is β. In the fault- and event tree modelling we may introduce extra basic events to represent the situation where two or more event occurs simultaneously due to common cause failures.

The challenge is to assess the numeric value of β. Podofolini et. al. (2009) summarizes the literature both with respect to factors included, and their importance. They find that the following factors are considered most important:

- Closeness in time
- Similarity of crew/performer(s)
- Stress
- Complexity

To be consistent with the Risk_OMT modelling framework we introduce a quantitative approach where each factor has a weight, say w_i and a score s_i, and where the resulting common cause factor is given by:

$$\beta = \beta_0 \prod_i w_i^{s_i} \tag{3.20}$$

Here β_0 is a baseline common cause factor. The scores are measured on a scale from $s = -1$ representing the best value we can imagine for the factor, and $s = 1$ the worst value we can imagine.

Table 3.1 presents numeric values for the weights used in Risk_OMT. Further principles for assigning a value to the score is given. Some of the scores are determine based on already identified RIFs, whereas other scores are determined by an evaluation of the type of tasks to be executed, or other conditions to be considered related to the actual situation.

The baseline dependency level is set to $\beta_0 = 0.05$ for failure types “violation”, “omission”, and “mistake”. For “slips & lapses” the common cause problem is considered slightly lower, and the value $\beta_0 = 0.03$ is recommended.

There are two feasible ways to include the common cause effects in the modelling. One way is to model explicitly the common cause effects by introducing additional basic events in the fault and event trees. For a full BBN model this is the only way to represent such common cause effects. If the hybrid approach with a mixture of BBN models and event and fault trees is

Table 3.1: Weights and principles for setting scores in the CCF model

Dependency factor	**Weight**	**Scores**	
		Best = -1	**Worst = 1**
Closeness in time	2	The closeness in time is assumed to depend on the type of tasks considered. The following scores are proposed for the relevant situations. Control planning | Planning, S = 1 Control | Execution, S = -1 Execution | Execution, S = ½ Note, we assume there are no dependencies between planning activities and execution activities.	
Similarity of crew	3	Different crew, S = -1	Same crew, S = 1
Stress	2	The stress level is based on the RIF for *time pressure.* Let r denote the linear mapping of the RIF on the interval (0,1) where A corresponds to 0, and F corresponds to 1. The score of the stress dependency factor is then given by $S = 2(r-½)$	
Complexity	1.5	The Risk_OMT model does not include any RIF explicitly used to describe complexity. The RIF for *design* and *HMI* are considered to be the most relevant RIFs indicating complexity. If the values of these are denoted r_1 and r_2 respectively, the score of the complexity dependency factor is then given by $S = (r_1 + r_2 - 1)$	

used, we may also introduce common cause failures in the post-processing of the minimal cut sets. The challenge then is to describe the possible dependencies for various classes of basic events, and then add common cause terms when the minimal cut set contributions are calculated. For example if a minimal cut set comprises the following basic events: {P=Planning error, CP=Control Planning error, E=Execution error, CE=Control Execution error} and we introduce $\beta_{\text{CP|P}}$ and $\beta_{\text{CE|E}}$ as common cause factors for controlling the plan, and controlling the execution respectively, we may use the following approximation to find the failure probability contribution from this minimal cut set:

$$Q_j \approx [q_{\text{P}} q_{\text{CP}} + \beta_{\text{CP|P}} \min(q_{\text{P}}, q_{\text{CP}})][q_{\text{P}} q_{\text{CP}} + \beta_{\text{CE|E}} \min(q_{\text{E}}, q_{\text{CE}})] \tag{3.21}$$

3.11 Importance measures

Risk importance measures are important in risk management. By having a good understanding of which factors and conditions contribute most to the risk, we may also start evaluating for which conditions risk reducing measures would be most efficient.

A common importance measure is the Birnbaums measure, $I^{\mathrm{B}}(i)$. For a fault tree or a reliability block diagram the measure is essentially a sensitivity measure reflecting the increase in system reliability if a component is improved.

In our Risk_OMT framework we have two challenges

- We deal with fault and event tree, meaning that there is not only one "TOP event", but it will be several end consequences to consider in such a sensitivity analysis
- The objective is to consider the RIFs, and not the basic events. Hence we would like to measure the total risk reduction if the condition of a RIF is improved. However, since the RIFs are described by stochastic variables, it is not straight forward to specify what is meant by improving a RIF.

To cope with the situation of several end consequences in the event tree, we may introduce a numerical loss to each end consequences, and calculate the expected loss over all end consequences. The expected loss is then used as our risk measure when establishing our importance measure. In the Risk_OMT project the main focus was to focus on only one critical event in the event trees, hence we can simplify and only treat the frequency of that critical end event, say F.

To cope with the fact that the RIFs are only known in terms of the posterior distribution, it is proposed to investigate the impact of a change in the expected value of the RIF. The rationale is now that it might be possible to claim that a proposed measure will improve the condition of the RIF, but the only thing we may say is that the effect of the measure will be a change in the expected value.

Now let ΔE_j be a small change in the expected value of RIF number j by implementing a risk reduction measures. For example $\Delta E_j = 1/12$ means that we can improve the RIF by a "half mark", for example if we have a "C",we end up with something in the middle between "B" and "C".

Let π_j be the posterior distribution of RIF number j before any improvement measure is considered, and let π_j^{Δ} be the distribution we imagine with the improvement measure. The frequency of the critical end consequence will be a function of the distribution of our RIF under consideration, and the proposed improvement measure is:

$$I^{\mathrm{B}}_{\mathrm{RIF}}(j) = \frac{F(\pi_j^{\Delta}) - F(\pi_j)}{\Delta E_j} \tag{3.22}$$

The calculation of equation (3.22) requires π_j^{Δ} and π_j to be discretized.

To "change" a RIF is not straight forward. We approach this challenge differently for level one and level two RIFs.

For level one RIFs the posterior distribution is conditional on the corresponding level two RIF. We can therefore not just "change" one of the RIFs without impacting the other RIFs. In the assessment we now handle this by "disconnecting" the level one RIF under consideration. I.e., we calculate the unconditional pdf, say π_j, and then we assume that this RIF is independent of the other siblings and proceed with π_j as a single node in the quantification, i.e., calculation of $F(\pi_j^{\Delta})$ and $F(\pi_j)$.

For level two RIFs there will be not very efficient to change only the posterior distribution for this RIF. This is because the children of that RIF has historically been influenced by the "old" RIF, hence the data in terms of scores for the level one RIFs reflects the historical regime, and will have a "moment of inertia". The proposed workaround is just to add ΔE_j directly to the *score* of all the children of the level two RIF under consideration. $F(\pi_j^{\Delta})$ in equation (3.22) is then actually not calculated by a modified posterior distribution, but we have a new calculation regime for $F(\cdot)$ based on "modified" level one scores.

3.12 Parameter estimation

The RIM parameters of interest are:

- q_{L} = Lowest value for the basic event probability, i.e., when all RIFs have state A
- q_{H} = Highest value for the basic event probability, i.e., when all RIFs have state F
- w_j = Standardized weight of level 1 RIF number j influencing the basic event
- $V_{\mathrm{P},j}$ = Structural importance of parent of level 1 RIF number j
- $V_{\mathrm{S},j}$ = Variance of the score of RIF number j given the true underlying RIF value r

3.12.1 Variances

$V_{\mathrm{P},j}$ and $V_{\mathrm{S},j}$ represent "lack of deterministic" relations that in the end will represent variance in the observed scores. The aim is now to use observations to estimate $V_{\mathrm{P},j}$ and $V_{\mathrm{S},j}$. Using the double expectation rule it might be shown that

$$\mathrm{Var}\left(S_{c,i}|r_{p,i}=s_{p,i}\right)=V_{\mathrm{P},j}+V_{\mathrm{S},j} \tag{3.23}$$

where $S_{c,i}$ is the score of the i'th observation of level one RIF number j, $r_{p,i}$ is the true underlying value of the corresponding level two RIF (i.e., the parent), and $s_{p,i}$ is the score for the parent RIF

for the i'th observation. This means that we may find the variance of a level one RIF is we assume that the score of the parent RIF is equal to the true underlying value of the RIF.

Now, assume that we have n observations of child and parent RIFs over some time period. In this period the true underlying values of the RIFs might change. We may now estimate the variance in equation (3.23) by:

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(s_{c,i} - s_{p,i}\right)^2 \tag{3.24}$$

where $s_{c,i}$ is the score of level one RIF number j for the i'th observation, and $s_{p,i}$ is the score of the corresponding parent RIF.

We are not able to separate $V_{\mathrm{P},j}$ and $V_{\mathrm{S},j}$. This means that we need to make an expert judgement regarding their relative magnitude. It is proposed to set

$$V_{\mathrm{S},j} = 1/4 V_{\mathrm{P},j} \tag{3.25}$$

which gives

$$\hat{V}_{\mathrm{P},j} = 1/5\hat{\sigma}^2 \tag{3.26}$$

$$\hat{V}_{\mathrm{S},j} = 4/5\hat{\sigma}^2 \tag{3.27}$$

The expression for the variance in Equation (3.23) assumes that the parent score equals the parent value of the RIF. If the parent score deviates from the parent value, the estimate for the variance in Equation /3.24) would be larger. Hence to elicitate the variances we should use:

$$\hat{\sigma}^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(s_{c,i} - s_{p,i}\right)^2 = V_{\mathrm{P},j} + V_{\mathrm{S},j} + V_{\mathrm{S},p_j} \tag{3.28}$$

where V_{S,p_j} is the variance of the score of the parent of level 1 RIF j. Assuming $V_{\mathrm{S},p_j} = V_{\mathrm{S},j}$ gives:

$$\hat{V}_{\mathrm{P},j} = 2/3\hat{\sigma}^2 \tag{3.29}$$

$$\hat{V}_{\mathrm{S},j} = V_{\mathrm{S},p_j} = 1/6\hat{\sigma}^2 \tag{3.30}$$

Exercise 3.9

Use the double expectation rule to prove Equation (3.23). Use Monte Carlo simulation to verify the result leading to Equations (3.26) and (3.27).

Exercise 3.10

Use Monte Carlo simulation to verify the Equation (3.28) when also unertainty in the parent

score is considered.

3.12.2 Estimating q_L, q_H and w_j

Recall, that in Risk_OMT it is only the first level RIFs that influence the failure probability. Therefore, when estimating q_L, q_H and w_j we only use the scores on the first level RIFs. A better approach would have been to insert the expected values of the underlying RIFs based on a combination of the score and the parent RIF.

The functional relation between the RIFs (r_j's) and the basic event probability is given by:

$$q = q_L\left(\frac{q_H}{q_L}\right)^{w_1 r_1 + w_2 r_2 + \ldots} = q_L\left(\frac{q_H}{q_L}\right)^{\sum_j w_j r_j} \tag{3.31}$$

Now, inserting the score values for the RIFs and taking logarithm gives:

$$\ln q = \ln\left(q_L\left(\frac{q_H}{q_L}\right)^{\sum_j w_j s_j}\right) = \ln q_L + \sum w_j s_j \ln\left(\frac{q_H}{q_L}\right) \tag{3.32}$$

By letting $Y = \ln q$, $\beta_0 = q_L$, $\beta_j = w_j \ln\left(q_H/q_L\right)$ and $x_j = s_j$ we get

$$Y = \beta_0 + \sum_j \beta_j x_j = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots \tag{3.33}$$

which essential is a multiple linear regression model. Note that in eq. (3.33) we use x for the independent variable, but in the estimation we will go back to s to avoid mixing up with the number of errors later on in the estimation procedure.

For one combination of the score vector $\mathbf{s} = [s_1, s_2, \ldots s_r]$, there might be one or more observations. Typically there will be several observations if we have data on the basic event over a period of time where the score vector is assumed to remain constant, typically between surveys executed to assess the scores. We use i as an index to run through the relevant combinations of the score vector, and s_{ij} is the corresponding value of the score for level 1 RIF number j. Let x_i be the reported number of failures or human errors, and n_i the number of "trials", i.e., execution of the "basic" event. In the regression model we replace q with it's estimate $\hat{q}_i = x_i/n_i$. However, taking the logarithm on the left hand side will cause problems in case $x_i = 0$.

This might be overcome with an empirical Bayesian approach where the prior distribution over q is found by the sample mean and variance of $\hat{q}_i$. The beta distribution would be a conjugate prior. For the beta distribution with parameters α_0 and γ_0 it might be shown that if we

know the mean value, say μ, and standard deviation, say σ, we have:

$$\gamma_0 = \left(\frac{\mu(1-\mu)}{\sigma^2} - 1\right)(1-\mu) \tag{3.34}$$

$$\alpha_0 = \frac{\mu\gamma_0}{1-\mu} \tag{3.35}$$

In the procedure we will no replace μ and σ in eqs. (3.34) and (7.8) with the sample mean and standard deviation for $\hat{q}_i$.

The compleFte procedure for the estimation is now

1. Find a prior distribution for q by an empirical Bayesian approach where the parameters in the prior distribution are given eqs. (3.34) and (7.8)
2. For each observation find the Bayes estimate for q_i by calculating $\alpha_i = \alpha_0 + x_i$ and $\gamma_i = \gamma_0 + n_i - x_i$
3. Calculate the dependent variable, $y_i = \ln\left[\alpha_i/(\alpha_i + \gamma_i)\right]$
4. Let s_{ij} be the score of the jth first level RIF for observation i
5. The standard multiple linear regression model now reads $Y_i = \beta_0 + \beta_1 s_{i1} + \beta_2 s_{i2} + \ldots + \beta_r s_{ir} + \epsilon_i$ Use a standard least square estimation procedure to find the estimates $\hat{\beta}_j$.
6. Find the standardized weights by: $\hat{w}_j = \hat{\beta}_j / \sum_{j=1}^{r} \hat{\beta}_j$
7. Find the range for q by: $\hat{q}_{\mathrm{L}} = e^{\hat{\beta}_0}$ and $\hat{q}_{\mathrm{H}} = \hat{q}_{\mathrm{L}} e^{\sum_{j=1}^{r} \hat{\beta}_j}$.

3.13 Reefs in the sea

Reflect on what is the difference:

1. The shipwreck was due to an unknown reef in the sea
2. A high density of reefs in the sea gives more shipwrecks

The first situation is relevant if we are analysing data from accident and incident report system, or investigation reports after accidents. The individual events are analysed and causes behind the accident or incident is reported. When analysing investigation reports from shipwreck for example in Norway, we may use such data to find for example the rate of shipwreck caused by hitting a reef:

$$\hat{f}_{\mathrm{Reef}} = \frac{\text{Number of shipwrecks caused by hitting a reef}}{\text{Number of years for which we have data}} \tag{3.36}$$

Obviously, having accident rates split on the various failure causes would be important, and this might help us implement risk reducing measure. However, Equation (3.36) will not help us distinguishing between how important the risk influencing factor "reef" is (weight) vs how many reefs there are (score).

Investigation reports might also help us in understanding factors influencing the damage potential of accidents. Let x_j be a factor to consider, and let Y be the severity of an accident. A first order approach to try to understand the relation between the various factors that might affect an accident, and the severity we could establish a standard multiple linear regression model:

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \ldots \tag{3.37}$$

For example if x_1 is the distance from the bridge to the fire station, we might expect that $\beta_1 > 0$. Such a model can also get insight into the problem, and help us to identify risk reducing measures. For example we can have more dense fire stations, or at least have fire stations close to critical bridges.

Mohaghegh et al. (2009) discusses (i) Regression-based techniques, (ii) Formal probabilistic risk analysis techniques and (iii) BBN = Bayesian belief nets. In our context we emphasize that regression-based techniques could be very valuable, but they are usually not easy to apply in connection with formal probabilistic risk analysis techniques and BBN.

When we approach the question "A high density of reefs in the sea gives more shipwrecks" we would like to have a more explicit risk model utilizing formal probabilistic risk analysis techniques and BBN. For example we might establish fault- and event trees to analyse the seaward approach to ports where we both identify number of reefs as a RIF, but where we also consider use of echo sounder, availability of maps, competence, use of pilot etc. as important RIFs.

Note if a reef is an important RIF in a risk model like Risk_OMT, it means that one or more model parameter will become significantly worse as the value of the RIF becomes worse and worse. The *importance* of the RIF is independent of the *value* in this context. In the "failure statistics" approach the RIF is important due to the combination of may reefs and failure to avoid hitting them, i.e., the statistical analysis cannot discriminate between the number of reefs and the mitigating measures we have.

In order to estimate model parameters for e.g., Risk_OMT we need accident and incident reports, but also exposure information in terms of description of fairway, technical condition on each ship, log of all ships entering the port etc.

Also in this situation we might establish regression models, but now these regression models are used to establish relation between risk influencing factors and model parameters in our fault- and event tree.

3.14 Aspects of dynamic risk analysis

Reflect on what does “dynamic” mean?

a) Wikipedia: The dynamical system concept is a mathematical formalization for any fixed “rule” that describes the time dependence of a point’s position in its ambient space

b) Dynamic risk analysis is the process of updating existing risk analysis in light of new observations, new working practice, new systems installed etc., i.e., it is an update

c) Dynamic risk analysis is about to understand here and now what is happening, and use this to make here and now decisions regarding what is safe to do, is additional temporary barriers required? etc.

Chapter 4

Bayesian belief network

4.1 Introduction

A Bayesian belief network (BBN = Bayesian Belief Network) is a probabilistic graphical model that represents a set of variables and their conditional dependencies via a directed acyclic graph. Test

- A Bayesian belief network, for example, can represent the probabilistic relationships between diseases and symptoms
- Given symptoms, the BBN can be used to calculate the probability of the presence of various diseases.

The motivation behind the “method” is Bayes’ theorem. We start with the conditional probability rule:

$$\Pr(A|B) = \Pr(A \cap B)/\Pr(B) \text{and} \tag{4.1}$$

$$\Pr(B|A) = \Pr(A \cap B)/\Pr(A) \tag{4.2}$$

which leads to Bayes’ theorem:

$$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)} = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)} \tag{4.3}$$

Equation (4.3) says how we can “reverse” the situation. This is useful if, for example, we have a parametric model where time-to-failure, T, is exponential distributed with parameter λ, i.e., we have $f(t|\lambda) = 1/\lambda e^{-\lambda t}$. If we know λ, we can make probability statements with respect to T corresponding to $\Pr(B \text{ (i.e.} T)|A \text{ (i.e.} \lambda))$. While if we do not know λ, but have an observation for the failure time T (a "proof"), we can make a probability statement about the failure rate,

at $\Pr(A(\text{i.e. } \lambda)|B \text{ (i.e.} T))$ using Bayes' theorem. This is actually what is happening in Bayesian statistics, where the probability desnity function for the failure rate λ is updated when we get an observation, i.e., a failure time $T = t$.

To determine $\Pr(B)$ in the demonimator, we often use the law of total probability, which yields:

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\sum_i \Pr(B|A_i)\Pr(A_i)} \tag{4.4}$$

BBN can both be used to give probability statements about future events, and by "inverting" the equations, i.e., using Bayes' theorem, we can also update model parameters when we get observations (evidence).

The theory behind BBN can be difficult to understand, and especially the technical details. This means that we in limited degree can demonstrate BBN by hand calculation. However, there are computer tools that can make the necessary calculations where the user only specifies dependencies in a graphical tool, as well as probability statements and evidence (observations) at hand.

4.2 Definition

A Bayesian network is a directed acyclic graph (DAG) where the nodes represent stochastic variables and the edges (arrows) of the graph represent conditional dependencies.

If there is an arrow from node A to another node B, A is denoted a parent node of B, and B is a child of node A.

4.3 Product rule

A Bayesian network is a DAG, while a DAG is only a Bayesian network if the common distribution of node values can be written as a product of local (marginal) distributions to each node and their parent nodes, i.e., we require:

$$\Pr(X_1 = x_1, X_2 = x_2, \ldots, X_n = x_n) = \prod_{i=1}^{n} \Pr(X_i = x_i | X_j = x_j \text{ for all parentnodes } j \text{ to node } i) \tag{4.5}$$

If X_i has no parent nodes, then the probability distribution is *unconditional*. Furthermore, if X_i is observed, we say that it is an evident node.

The above product rule can in theory be used to make probability statements in the network, i.e., *inference*. To do this, we need the conditional probabilities. These are structured in so-called conditional probability tables (CPT = Conditional Proability Tables). The challenge with the

calculations is that there are a lot of combinations to calculate, and even when using computers, the calculation time will become almost infinitely long, unless you can use clever heuristics or approaches. The commercial tools have good algorithms for such calculations.

4.4 Conditional probability tables

To specify a BBN model, we need to specify conditional probabilities. This is done by use of so-called conditional probability tables. Note that a BBN model can have stochastic variables (nodes) that are both discrete and continuous. But in order to be able to carry out calculations it is common to approximate all quantities with discrete stochastic variables. In fact, binary variables are often used to reduce the calculation and specification time of the model.

A conditional probability table (CPT = Conditional Probability Table) is a table for a stochastic variable where conditional probabilities for each value the variable can be specified for all possible combinations for the parent nodes.

If X_i has no parent nodes, then the probability distribution is *unconditional*. Furthermore, if X_i is observed, we say that it is an evident node.

Example 4.1
It is reasonable to assume that the probability of a human error depends on whether the work can be carried out indoors or outdoors. We define the following variables:

- $X_1 = T$ (true) if human error, F (false) otherwise
- $X_2 = T$ if the work is carried out indoors, F otherwise

An associated CPT can then be:

Table 4.1: Conditional probability table for human error

Indors	Human error	
	T	F
T	0.001	0.999
F	0.005	0.995

In the example, there is only one parent node, i.e., “Indoors”, while in principle we can have several parent nodes. In this example, it is natural to think of the parent nodes as “influencing factors”, cf. the PSFs in the chapter on human error.

4.5 Fault tree represented as a BBN

Any fault tree can be represented as a BBN diagram. This can be useful if the basic events in the fault tree have dependencies that we want to model in more detail. We can then create BBN structures for the basic events, and then connect these together first in the fault tree, and then to convert to a BBN diagram where both the underlying structures and the "logic" of the fault tree are collected in a common BBN model. This is an elegant approach, but it has turned out that the BBN model quickly becomes too large to be handled effectively.

Figure 4.1 illustrates how a fault tree can be represented as a BBN diagram. The fault tree is to the left, and the corresponding BBN diagram is on the right:

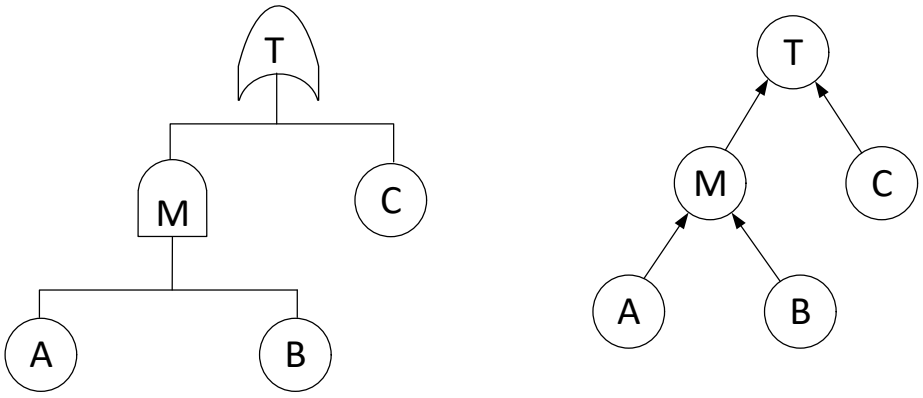


Figure 4.1: Fault tree to the left and BBN diagram to the right.

The TOP event is here indicated as T, M is an AND gate that connects A and B, while the TOP event T is an OR gate. In the corresponding BBN diagram, we do not have gates, and the logical structure will therefore be represented by CPTs, which are referred to here as "truth tables". A truth table is a binary table, where 1 means "True" and 0 means "False". Since this is a fault tree, "True" means that a gate *occurs*, while "False" means that it does not occur.

Figure 4.2 shows truth table for M in the BBN:

Table 4.2: Truth table for M

A	B	M | A, B
0	0	0
0	1	0
1	0	0
1	1	1

We see that M is true only if both A and B are true, i.e., that both basic events A and B occur corresponding to the AND gate. Figure 4.3 shows the truth table for T:

Table 4.3: Truth table for T

C	M	T | C, M
0	0	0
0	1	1
1	0	1
1	1	1

Note that nodes A, B, and C have no parent nodes. This means that they are stochastically *independent*. The CPTs for these then only have two values, i.e., one for true, and one for false. Here, true will correspond to the probability that the basic event occurs (e.g., a failure), while false is the probability that the basic event is not occuring. Note that the truth tables are *regular* CPTs, so the calculation algorithms can be applied directly by specifying all the tables in a BBN tool.

Chapter 5

Human reliability analysis

5.1 Introduction

The purpose of a human reliability analysis (HRA) is to analyse human performance of sociotechnical systems. There are several objectives to conduct an HRA analysis such as ensure that humans are able to conduct their main tasks in operation of sociotechnical systems in a safe and reliable manner, to identify measures to improve human performance, and to quantify human errors as part of assessing risk of critical operations. One of the first systematic attempts to quantify human error probabilities (HEPs) was the Technique for Human Error Rate Prediction (THERP, Swain & Gutmann, 1983). A main purpose of the THERP method was to give input with respect to human performance into technical risk analyses. The methodological achievement of HRA techniques has primarily been achieved in the nuclear industry, but HRA methods have also extensively been applied in high risk activities such as aviation and the oil and gas industry. The THERP method is considered to be time consuming from an analysis point of view, hence simplified methods have been proposed. Among these are the Human Error Assessment and Reduction Technique (HEART, Williams, 1992) and more recently the Standardized Plant Analysis Risk (SPAR-H) human reliability analysis method (Gertman et. al., 2005). The SPAR-H methodology will be presented in later sections. Many HRA quantification techniques have been criticized because they treat humans in the same way as components and do not pay sufficient attention to the cognitive performance of humans. Therefore also Cognitive control based techniques have been proposed, among them is the Cognitive Reliability and Error Analysis Method (CREAM, Hollnagel 1998). In 2007 the HSE in UK presented a survey of HRA techniques for which 17 techniques were given a recommendation (Bell and Holroyd, 2009).

The Petro-HRA method was developed in the R&D project "Analysis of human actions as barriers in major accidents in the petroleum industry, applicability of human reliability analysis methods". The project was supported by the Norwegian Research Council and conducted during the period 2012-2016. The main result from the project was a guideline for the Petro-HRA

method. The guideline has later been developed and the latest version is from 2022. The Petro-HRA method is mainly based on the SPAR-H method, but adapted to the petroleum industry.

Three important definitions are listed as a basis for the outline of the HRA techniques:

Human error: An out-of-tolerance action, or deviation from the norm, where the limits of acceptable performance are defined by the system. These situations can arise from problems in sequencing, timing, knowledge, interfaces, procedures, and other sources (NUREG/CR-6883).

Human error probability (HEP): A measure of the likelihood that plant personnel will fail to initiate the correct, required, or specified action or response in a given situation, or by commission will perform the wrong action. The HEP is the probability of the human failure event (ASME RA-S-2002).

Performance shaping factor (PSF): A factor that influences human performance and human error probabilities is considered in the HRA portion of risk analysis (NUREG/CR-6883).

In the RiskOMT method the term HEP is not used, rather the failure probability of basic events q, in a quantitative risk analysis or a barrier analysis is used. The RiskOMT term risk influencing factor, RIF, is used rather than PSF. Note that PSF's corresponds to level one RIFs in the RiskOMT method.

5.1.1 Main steps of an HRA

A typical quantitative HRA comprises the following steps (Rausand, 2011):

1. Identify critical operations where human errors could lead to accidents and/or operational problems. Such operations could be a critical marine operation as erecting the tower of a wind turbine, accessing an offshore energy converter, or conducting maintenance of a safety critical system.

2. Analyse relevant tasks and break down to subtasks and task steps. Various types of task analysis (Kirwan and Ainsworth, 1992) are often used. In subsequent sections the hierarchical and tabular task analysis will be presented.

3. Identify human error modes and, if possible, error causes and performance-influencing factors. Relevant techniques are AEMA, Human HAZOP and the tabular task analysis method presented in subsequent sections.

4. Determine human error probabilities (HEPs) for error modes and for a complete task by combining result for individual tasks with a sequence of tasks analysed by e.g., event tree analysis (ETA).

Task analysis may comprise the following main steps:

1. Breakdown of the task into subtasks and simple task steps.
2. Description of the allocation of task steps between different persons. This description gives an indication of communication needs.
3. Description of the temporal dependencies between subtasks or task steps. This will give input to quantification of so-called common cause failures.
4. Classification of task types (and task step types).
5. Identification of cues and feedback supporting each task step. The cues indicate to the operator that an action can/should be initiated (e.g., a *red traffic light* informs the driver to stop). The feedback informs the operator about the effects of carrying out the action (e.g., the driver will feel the *retardation* knowing that breaking takes place).

In order to get a better understanding of task types Rasmussen (1979) proposed to classify behaviour into the so-called Skill, Rule and Knowledge-based behaviour regime:

- **Skill-based**. Subconscious, automated actions requiring little or no cognitive effort (e.g., speed regulation in curves during ordinary driving).
- **Rule-based**. Actions according to an explicit rule or procedure (e.g., stopping at red light).
- **Knowledge-based**. Coping with unfamiliar situations without a procedure (e.g., diagnosis) requires conscious problem solving and decision-making.

Later Reason (1990) proposed a categorisation of human errors into slips, lapses, mistakes and violations:

- **Slip**. An action that is carried out with a correct intention but a faulty execution.
- **Lapse**. A failure to execute an action due to a lapse of memory or because of a distraction.
- **Mistake**. A correct execution of an incorrect intention.
- **Violation**. A person deliberately applies a rule or procedure that is different from what she knows is required, even though she may often do it with good intent.

Such categories may be of value when analysing tasks and possible human errors. The task analysis of a critical activity is often conducted in two steps. First the relations between tasks are visualised in a hierarchy (HTA = Hierarchical Task Analysis). On the TOP level the main task is shown. Then the necessary sub-task necessary for accomplishing the main task is shown with a plan. The plan shows the sequence of sub-task, typically *do in any order*, or *1 and 2 in order, then IF <condition> DO 3*. Each sub-task may then be further broken down into sub-sub tasks and so on. Figure 5.1 shows an example of a hierarchical task analysis for accessing a wind turbine from a boat.

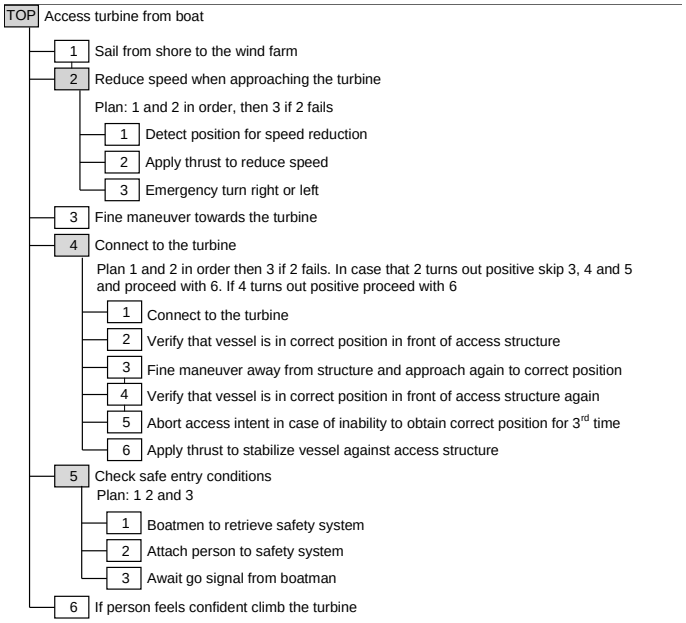


Figure 5.1: Snapshot of a hierarchical task analysis for the task of assessing the wind turbine from a boat

The tasks on the lowest level are now analysed by a tabular task analysis (TTA). Alternatively a Human HAZOP or an AEMA (Action Error Mode Analysis, Rausand, 2011) may be conducted. The purpose of the tabular task analysis is to identify possible human error (HEI = Human Error Identification). To structure the analysis each task from the HTA is analysed in one row in the TTA worksheet. In the following the term *action* is used to denote the task on the bottom level of the hierarchical task breakdown. Standard columns in a TTA comprise:

- ID
- Action
- Cues
- Feedback

- Possible errors (Action errors)
- Error causes
- Comments

The TTA does not usually provide guide-words for identification of possible errors. But we may adopt guide-words from techniques like Human HAZOP or AEMA. Like the standard HAZOP a set of guide-words are used to identify action errors. Typical guide-words are then (Shorrock et al. 2003)

- No action
- More action
- Less action
- Wrong action
- Part of action
- Extra action
- Other action
- More time
- Less time
- Out of sequence
- More information
- Less information
- Wrong information

Combining the action with the guide-word gives possible action errors. For example the action “Open valve” combine with “Other action” gives the action error “Open wrong valve”. Note that the error causes will be considered later when assessing the effect of performance shaping factors (PSF) in the quantification part of the analysis.

5.1.2 Quantification of HEPs for action and complete tasks

A range of quantification methods have been proposed in order to asses human error probabilities. In the following the SPAR-H method is presented. The SPAR-H method is one of the recommended HRA techniques by HSE in UK (Bell and Holroyd, 2009), and is also partly based on three other methods (THERP, HEART and CREAM). The fact that it is also rather easy to implement and to apply resulted in that this method was chosen for presentation here. Although it is recommended to read the SPAR-H report the following presentation will give an idea how the method works.

SPAR-H - Introduction

SPAR-H presents a simple method for assessing the human error probabilities associated with operator and crew actions and decisions in response to initiating and pre-initiator events. The method has been developed for use in the nuclear industry, and some adaptions have been made in this presentation. The basic SPAR-H framework comprises the following:

- Decomposes probability into contributions from diagnosis failures and action failures
- Accounts for the context associated with human failure events (HFEs) by using performance-shaping factors (PSFs), and dependency assignment to adjust a base-case HEP
- Uses pre-defined base-case HEPs and PSFs, together with guidance on how to assign the appropriate value of the PSF
- Employs a beta distribution for uncertainty analysis (not covered in this presentation)
- Uses designated worksheets to ensure analyst consistency.

An appealing feature of the SPAR-H method is that only two generic failure modes are considered, i.e., diagnosis failures and action failures. This is in contrast to methods like CREAM and HEART that introduces several generic task types[1]. To compensate for the limited number of task types and failure modes the performance shaping factors therefore have a relative strong impact. Also, compared to other methods, SPAR-H introduces only eight PSFs making the method fast compared to other methods where up to 40 factors are considered.

SPAR-H – Generic task types and baseline HEPs

SPAR-H introduces two task types with corresponding error modes:

Diagnosis failures: These are failures with respect to correct diagnosis of a situation. The diagnosis failure mode is also used as a failure mode as part of planning activities and verifications.

Action failures: These are failures in physical execution of work.
The following baseline (nominal) human error probabilities are given for these failure modes:

- Diagnosis failures: Nominal HEP = 10^{-2}
- Action failures: Nominal HEP = 10^{-3}

The baseline HEPs of SPAR-H have been developed for the nuclear industry where tasks are conducted indoor. In situations where most activities will be outdoor activities sometimes conducted under harsh environmental conditions. Since SPAR-H do not provide PSFs for workplace conditions, it is therefore proposed to introduce a more comprehensive set of nominal HEPs (NHEP) shown in Table 5.1:

Table 5.1: Modified nominal HEPs (NHEP) from SPAR-H adjusted for indoor/outdoor applications (Vatn et al., 2012)

Failure mode	NHEP
Diagnosis failure – Outdoor, normal weather	2×10^{-2}
Action failure – Outdoor, normal weather	3×10^{-3}
Diagnosis failure – Outdoor, bad weather	5×10^{-2}
Action failure – Outdoor, bad weather	10^{-2}
Diagnosis failure – Indoor	10^{-2}
Action failure – Indoor	10^{-3}

SPAR-H – Performance shaping factors (PSFs)

SPAR-H introduces eight PSFs which are:

1. Available time
2. Stress and stressors
3. Experience and training
4. Complexity
5. Ergonomics (including the human-machine interface)
6. Procedures

7. Fitness for duty
8. Work processes

For a detailed description and discussion of these factors reference is made to the SPAR-H report (Gertman et.al., 2005). For each activity for which a HEP is to be assigned, a review of the state of the PSFs is required. A dedicated worksheet is provided, but the essence of this worksheet is shown in Table 5.2 for *diagnosis* failure modes and in Table 5.3 for *action* failure modes.

SPAR-H - HEP correction based on composite PSF correction factors

The factors for each PSFs are multiplied to give a composite factor, say $F = f_1 \cdot f_2 \cdot \ldots \cdot f_8$, yielding the adjusted HEP:

$$\text{HEP} = F \cdot \text{NHEP} = f_1 \cdot f_2 \cdot \ldots \cdot f_8 \cdot \text{NHEP} \tag{5.1}$$

In case of more than 3 negative HEPs (factor > 1) an adjustment formula is proposed to avoid probabilities larger than one:

$$\text{HEP} = \frac{F \cdot \text{NHEP}}{(F-1) \cdot \text{NHEP} + 1} \tag{5.2}$$

Note that a correction factor of "Pr(Failure)=1" in Table **8** and Table 9 means that the entire failure mode probability is set to one for this failure mode.

SPAR-H Dependency calculations

When activities are conducted in a sequence, it is generally believed that a failure of one activity will lead to a higher failure probability of subsequent activities due to common mode errors. To assess conditional probability for subsequent activities a dependency assessment is carried out. Four factors are considered for activities that may be dependent on previous activities in the course of events:

- Crew (*s*=same or *d*=different)
- Time (*c*=close in time or *nc*=not close in time)
- Location (*s*=same or *d*=different)
- Cues (*a*=additional or *na*=no additional)

Now, let the PSF Without Formal Dependence be denoted HEPw/od and assume that the dependency has been categorised according to Table 5.4. Further let HEPw/d denote the probability of failures With Formal Dependence. HEPw/d is now found form Table 5.5.

Table 5.2: Correction factors for different PSF levels for each PSF for diagnosis

PSF	PSF level	Factor
Available Time	Inadequate time	Pr(Fail.)=1
	Barely adequate time (approx. 2/3 x nominal)	10
	Nominal time	1
	Extra time (between 1 and 2 x nominal & > 30 min)	0.1
	Expansive (between 2 and 4 x nominal & > 30 min)	0.05
	Expansive (> 4 x nominal & > 30 min)	0.01
	Insufficient Information	1
Stress/Stressors	Extreme	5
	High	2
	Nominal	1
	Insufficient Information	1
Complexity	Highly complex	5
	Moderately complex	2
	Nominal	1
	Obvious diagnosis	0.1
	Insufficient Information	1
Experience/Training	Low	10
	Nominal	1
	High	0.5
	Insufficient Information	1
Procedures	Not available	50
	Incomplete	20
	Available, but poor	5
	Nominal	1
	Diagnostic/symptom oriented	0.5
	Insufficient Information	1
Ergonomics/HMI	Missing/Misleading	50
	Poor	10
	Nominal	1
	Good	0.5
	Insufficient Information	1
Fitness for Duty	Unfit	Pr(Fail.)=1
	Degraded Fitness	5
	Nominal	1
	Insufficient Information	1
Work Processes	Poor	2
	Nominal	1
	Good	0.8
	Insufficient Information	1

Table 5.3: Correction factors for different PSF levels for each PSF for action

PSF	PSF level	Factor
Available Time	Inadequate time	Pr(Fail.)=1
	Time available is approx. the time required	10
	Nominal time	1
	Time available >= 5x the time required	0.1
	Time available is >= 50x the time required	0.01
	Insufficient Information	1
Stress/Stressors	Extreme	5
	High	2
	Nominal	1
	Insufficient Information	1
Complexity	Highly complex	5
	Moderately complex	2
	Nominal	1
	Insufficient Information	1
Experience/Training	Low	3
	Nominal	1
	High	0.5
	Insufficient Information	1
Procedures	Not available	50
	Incomplete	20
	Available, but poor	5
	Nominal	1
	Insufficient Information	1
Ergonomics/HMI	Missing/Misleading	50
	Poor	10
	Nominal	1
	Good	0.5
	Insufficient Information	1
Fitness for Duty	Unfit	Pr(Fail.)=1
	Degraded Fitness	5
	Nominal	1
	Insufficient Information	1
Work Processes	Poor	5
	Nominal	1
	Good	0.5
	Insufficient Information	1

Table 5.4: Assignment of dependency level

Crew	Time	Location	Cues	Dependency
s	c	s	na	complete
			a	complete
		d	na	high
			a	high
	nc	s	na	high
			a	moderate
		d	na	moderate
			a	low
d	c	s	na	moderate
			a	moderate
		d	na	moderate
			a	moderate
	nc	s	na	low
			a	low
		d	na	low
			a	low

SPAR-H Event Sequence Diagram

In order to model the sequence of tasks established in the HTA (see Figure 5.1) an event tree, cause consequence diagram or event sequence diagram is used. Figure 5.2 shows a generic example of such a diagram. To find the end event frequencies the frequency of the initiating (hazardous) event is multiplied with the success or failure probabilities along the path leading to the end event in Figure 5.2. Note that conditional dependency probabilities are required, i.e., the HEPw/d-values from Table are required.

For example HEART introduces the following generic task types: 1) Totally familiar, performed at speed with no idea of likely consequences, 2) Shift or restore system to new or original state on a single attempt without supervision or procedures, 3) Complex task requiring a high level of comprehension and skill, 4) Fairly routine task performed rapidly or given scant attention, 5) Routine highly practised, rapid task involving a relatively low level of skill, 6) Restore or shift a system to the original or new state following procedures with some checking, 7) Completely familiar, well-designed, highly practised routine task occurring several times per hour, 8) Respond correctly to system command even when there is an augmented or automated supervisory system.

Table 5.5: Correction formulas for HEPs with formal treatment of dependencies

Dependency level	Correction formulas for dependencies
Complete	HEPw/d = 1.
High	HEPw/d = (1+ HEPw/od)/2
Moderate	HEPw/d = (1+6HEPw/od)/7
Low	HEPw/d = (1+19HEPw/od)/20
Zero	HEPw/d = HEPw/od

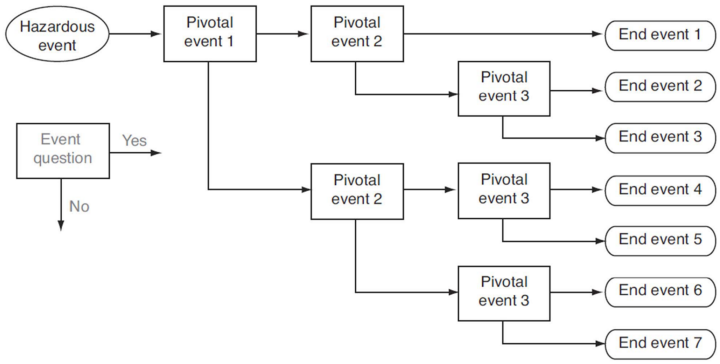


Figure 5.2: Event sequence diagram

Human error classification and probabilities

This section elaborate more on human error classification and human error probabilities. In particular we review aspects of human error classification used in the Risk_OMT model.

Human error is defined by Reason (1990): *The failure of planned actions to achieve their desired ends - without the intervention of some unforeseeable event.* This definition does not really elaborate on different aspects, so therefore a classification regime is often introduced.

5.1.3 Failure of omission and failure of execution

A failure of an activity may further be divided into failure of omission and failure of execution. Failure of omission denotes whether or not the prescribed activity is carried out. Failure of execution denotes inadequate actions that may cause failures, e.g., acts performed in a wrong sequence, at the wrong time, without the required precision, etc.

Failure of execution is seen as results of violations or human errors (Reason, 1990). Viola-

tions refer to both deliberate and unintentional omissions of one or several steps within a work task.

Human error is further divided into mistakes and slips & lapses, where mistakes involve actions that are based on failure of interpretation of procedures, and/or failures of judgemental/inferential processes involved in the prescribed activity. Slips & lapses involve actions that represent unintended deviation from those practices represented in the formal procedures. To summarize we have for the failure of an activity:

- Omission failure
- Execution failure
  - Violation failure
  - Human error
    - Slips & lapses
    - Mistakes

Figure 5.3 shows the corresponding fault tree with the RIF structure. Note that in Risk_OMT the RIF structure is only developed for execution failures.

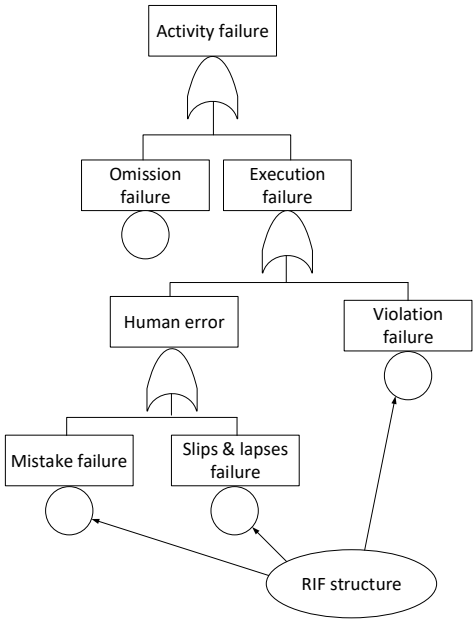


Figure 5.3: Structuring activity failure and RIF structure

5.1.4 Generic RIF model for execution and control activities

In Risk_OMT a generic RIF structure is developed. The level two RIFs are management RIFs related to:

- Competence
- Information
- Technical issues
- General issues
- Tasks

For each level two RIF there is one more level one RIFs. In In Risk_OMT there is one generic model for execution and control activities as shown in Figure 5.1.4, and one generic model for planning activities shown in Figure 5.5. This means that the type of activity determines the qualitative nature of the RIF structure.

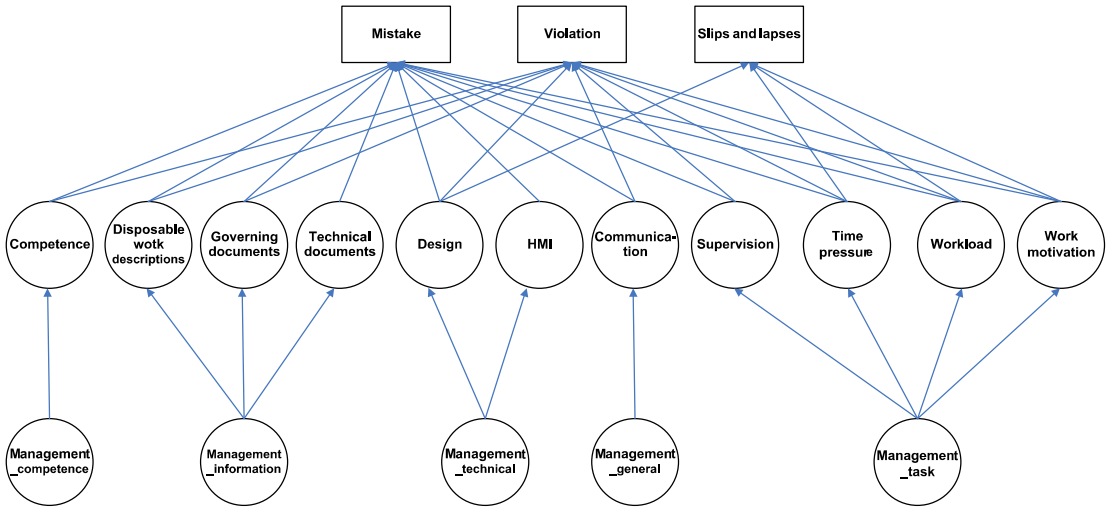


Figure 5.4: Generic RIF model for execution and control activities

5.1.5 Weights and "variances"

Note the following:

- In Risk_OMT the basic event failure probabilities depend on the *value* of each RIF, and the relative weight of the RIFs

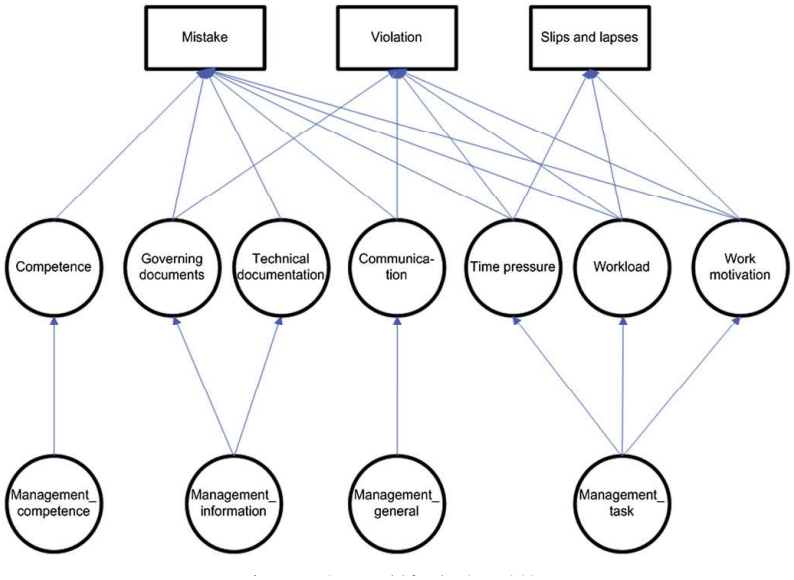


Figure 5.5: Generic RIF model for planning activities

- It is assumed that a particular RIF, e.g., "Technical documentation" is the same for all basic event. This means that we assess the values of the RIFs independent of which basic event is considered.
- The weights of the RIFs could in principle be different for each and every basic event. However, it might be more efficient to give a set of weights for a given type of basic events.
- Referring to Figures 2 and 3, we should as a minimum define 6 set of weights, i.e., for "mistakes", "violations", "slips & lapses" for both "planning" activities and "execution and control" activities.
- In Risk_OMT we define two set of "variances". For all RIFs we need to define the variance of the score given the true underlying RIF. This variance represents how difficult it is to get to know the underlying RIF by the information we have. The second variance is the structural variance, i.e., how much variance it will be in the true value of a first level RIF given a value of the corresponding second level RIF.
- The assessment of these two type of variances are done RIF by RIF, and is common for all basic events.

5.1.6 Nominal human error probabilities

In human reliability analysis we distinguish between human error probabilities (HEPs) and nominal HEPs. The HEPs are the probabilities we will use in a given analysis for a given set of the RIFs assessed. The nominal HEPs are the baseline HEPs. In Risk_OMT and BORA these nominal HEPs are those values for the HEPs we will have in the "average" case, i.e., if all RIFs were on their average, corresponding typically to a C.

In the BORA papers numerical values for nominal HEPs are discussed. The proposed numerical values are those that by the authors are considered relevant for oil and gas, and the type of tasks that are involved in typical maintenance and operation activities. It might be relevant to use these numerical values as starting point also for similar activities in maintenance and operation of e.g., infrastructure systems. However, for tasks that are of more "crisis management" and sharp end operation of equipment it might be required to do further investigation.

In the Risk_OMT papers some more considerations are made to take into account the more detailed model for categorizing the activity failures.

5.1.7 Error factors

In the Risk_OMT an error factor is used to define the spread of the HEPs relative to the nominal HEPs. The error factors are discussed in the Risk_OMT papers, but no numerical values are given. In the BORA papers the error factors are given for some type of activities, but though not for the different failure categories. Typical values of the error factors are in the order of magnitude 3 to 5.

Chapter 6

Siutation awareness

6.1 Introduction

Situational awareness or situation awareness (SA) is the understanding of an environment, its elements, and how it changes with respect to a time vector or other factors, is critical for appropriate and optimized decision making in many environments.

Several authors are using the term, and both "situational" and "situation" are used when referring to SA. Endsley (1995) is the most cited paper, and she defines SA as:

Situational awareness: The perception of the elements in the environment considering time and space, the understanding of their meaning, and the prediction of their status in the near future.

Several authors have criticized the framework proposed by Endsley (1995) and in a paper from 2015 she claim that most of the critique is based on misunderstanding of her original paper, see Endsley (2015).

This chapter in the course compendium is to a large extend based on the Wikipedia article on SA.

6.2 Endsley's Cognitive Model of SA

The most widely cited and accepted model of SA was developed by Endsley (1995) which has been shown to be largely supported by research findings.

Endsley's model describes the cognitive processes and mechanisms that are used by people to assess situations to develop SA, and the task and environmental factors that also affect their ability to get SA. It describes in detail the three levels of SA formation: perception, comprehen-

sion, and projection. Figure 6.1 shows the model:

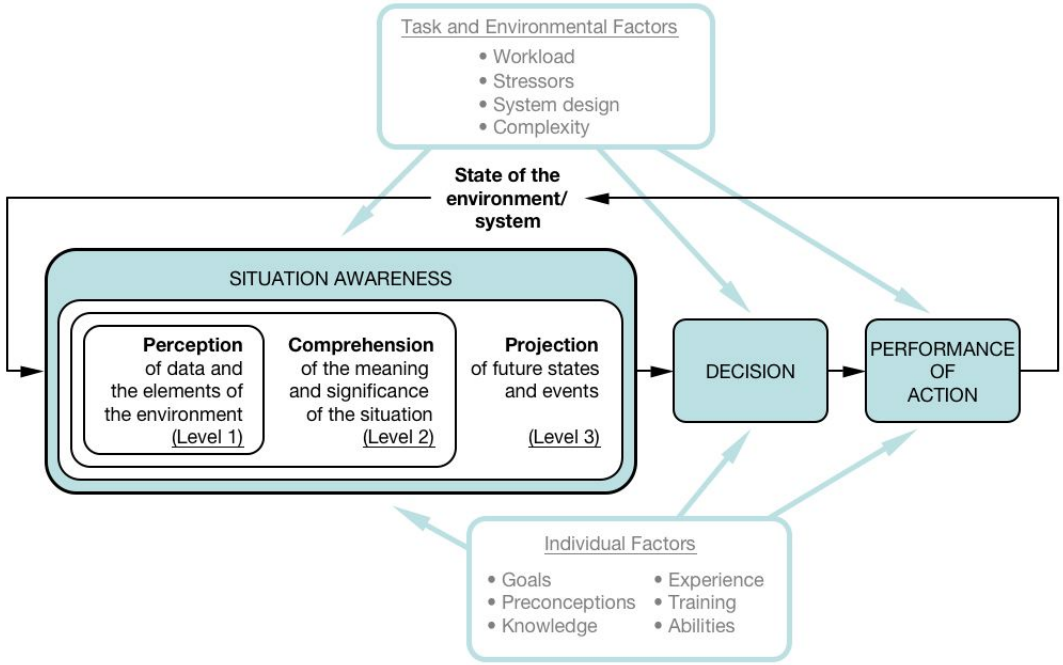


Figure 6.1: Endsley's model of SA, (Endsley, 1995)

The next sections describe the core content of the three levels of SA. Some authors have argued that these levels are to be interpreted as linear development of SA, but Endsley (2015) argues that this is not the case in her paper form 2015.

- *Perception (Level 1 SA):* The first step in achieving SA is to perceive the status, attributes, and dynamics of relevant elements in the environment. Thus, Level 1 SA, the most basic level of SA, involves the processes of monitoring, cue detection, and simple recognition, which lead to an awareness of multiple situational elements (objects, events, people, systems, environmental factors) and their current states (locations, conditions, modes, actions).

- *Comprehension (Level 2 SA):* The next step in SA formation involves a synthesis of disjointed Level 1 SA elements through the processes of pattern recognition, interpretation, and evaluation. Level 2 SA requires integrating this information to understand how it will impact upon the individual's goals and objectives. This includes developing a comprehensive picture of the world, or of that portion of the world of concern to the individual.

- *Projection (Level 3 SA):* The third and highest level of SA involves the ability to project the future actions of the elements in the environment. Level 3 SA is achieved through knowledge of the status and dynamics of the elements and comprehension of the situation

(Levels 1 and 2 SA), and then extrapolating this information forward in time to determine how it will affect future states of the operational environment.

6.2.1 Decision making

Figure 6.1 depict SA as the primary basis for subsequent decision making and performance in the operation of complex, dynamic systems. The feedback loop (State of the environment/system) is important, and a distinction could be made between:

- Decisions to gain more knowledge regarding the state of the environment and system, e.g., collect more information, open a new window on the computer, run simulations etc.
- Decisions to change the system state

See Endsley (2015) for further discussions.

Time and space

SA also involves both a temporal and a spatial component. Time is an important concept in SA since as new inputs enter the system, the individual incorporates them into this mental representation, making changes as necessary in plans and actions in order to achieve the desired goals. SA also involves spatial knowledge about the activities and events occurring in a specific location of interest to the individual.

Endsley's model of SA illustrates several variables that can influence the development and maintenance of SA, including individual, task, and environmental factors.

Key factors that describe the cognitive processes involved in SA are according to Endsley (1995).:

- Perception, comprehension, and projection as three levels of SA,
- The role of goals and goal directed processing in directing attention and interpreting the significance of perceived information,
- The role of information salience in "grabbing" attention in a data-driven fashion, and the importance of alternating goal-driven and data-driven processing,
- The role of expectations (fed by the current model of the situation and by long-term memory stores) in directing attention and interpreting information,
- The heavy demands on limited working memory restricting SA for novices and for those in novel situations, but the tremendous advantages of mental models and pattern matching to prototypical schema that largely circumvent these limits,

- The use of mental models for providing a means for integrating different bits of information and comprehending its meaning (relevant to goals) and for allowing people to make useful projections of likely future events and states,
- Pattern matching to schema - prototypical states of the mental model - that provides rapid retrieval of comprehension and projection relevant to the recognized situation and in many cases single-step retrieval of appropriate actions for the situation.

The model also points to a number of features of the task and environment that affect SA:

- The capability of the system and the user interface for conveying important information to the person in a way that is easy to integrate and process.
- Both high workload and stress can negatively affect SA. Information overload is a problem in many situations.
- Underload (vigilance conditions) can also negatively affect SA.
- The complexity of the systems and situations a person is in can negatively affect SA by making it difficult to form accurate mental models.
- Automation is a major factor reducing situation awareness in many environments (e.g. aviation, driving, power operations). See out of the loop performance problems. This is due to it creating situations where people are forced to become monitors which they are poor at (due to vigilance problems), often poor system transparency with needed information not provided, and an overall reduction in the level of cognitive engagement of people with automated systems

Experience and training have a significant impact on people's ability to develop SA, due to its impact on the development of mental models that reduce processing demands and help people to better prioritize their goals.

A note should be made regarding use of *schemata* and automation. In psychology and cognitive science, a *schema* describes a pattern of thought or behaviour that organizes categories of information and the relationships among them. It can also be described as a mental structure of preconceived ideas, a framework representing some aspect of the world, or a system of organizing and perceiving new information, such as a mental schema or conceptual model. Schemata influence attention and the absorption of new knowledge: people are more likely to notice things that fit into their schema, while re-interpreting contradictions to the schema as exceptions or distorting them to fit. Schemata have a tendency to remain unchanged, even in the face of contradictory information. Schemata can help in understanding the world and the rapidly changing environment. People can organize new perceptions into schemata quickly as

most situations do not require complex thought when using schema, since automatic thought is all that is required.

6.2.2 Relationship of goals and mental models to SA

Endsley (1995) points out that SA is an ongoing dynamic process of gathering and interpreting information to update the situation model and using that situation model to search for information until decisions can be made. Further there is a linkage between goals and mental models that drives the development or selection of plans and scripts for directing actions, the use of the activated mental model to direct attention to the environment to feed into the constantly updated situation model, and use of the situation model in updating the selection of the mental model to be active. Figure 6.2 depicts relationship of goals and mental models to SA.

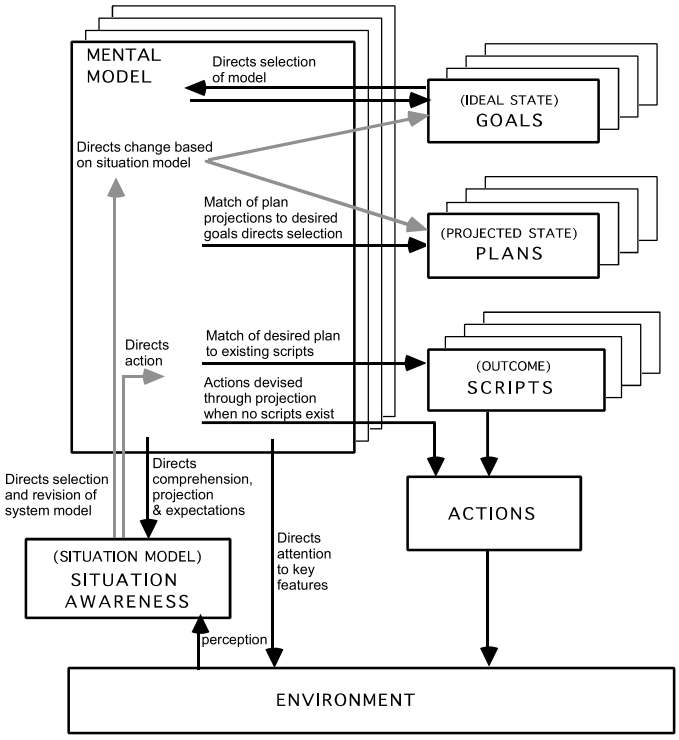


Figure 6.2: Relationship of goals and mental models to SA (Endsley, 1995)

As indicated above, we could distinguish between two types of decisions important to understand the feedback loop in Figure 6.1:

- Decisions to gain more knowledge regarding the state of the environment and system, e.g., collect more information, open a new window on the computer, run simulations etc.
- Decisions to change the system state

In Figure 6.2 there are three “feedback” loops:

1. ENVIRONMENT →(perception)→ SIUTATION AWARENESS →(Directs action)→MENTAL MODEL→(Directs attention to key features)→ENVIRONMENT

2. ENVIRONMENT →(perception)→ SIUTATION AWARENESS →(Directs action)→MENTAL MODEL→(Match of desired plan to existing scripts)→SCRIPT→ ACTION →ENVIRONMENT

3. ENVIRONMENT →(perception)→ SIUTATION AWARENESS →(Directs action)→MENTAL MODEL→(Actions devised thorough projection when no scripts exist)→ACTION→ENVIRONMENT

Chapter 7

Time modelling

7.1 Introduction

In the Risk_OMT hybrid model the RIF's are assumed to influence the basic event probabilities modelled in the fault tree. The focus is on explicit events defined, for example fail in i) Writing work order description, ii) Verifying work order description, iii) Requesting a work order permit etc. The time dimension was not emphasized explicitly, the only aspect of time included was "time pressure".

In other situations it is natural to consider time explicitly. As a motivating example consider the shift supervisor on a ship where there is another ship on collision course from port side. Further consider situation awareness levels according to Endsley (1995):

- *Perception (Level 1 SA):* This level involves the processes of monitoring, cue detection, and simple recognition, which lead to an awareness of multiple situational elements. We will denote these elements "signals". In the example this could be sound, visual information, information from technical systems etc. Essentially we are talking about getting attention to the situation of the other ship approaching.

- *Comprehension (Level 2 SA):* This step involves a synthesis of disjointed Level 1 SA elements through the processes of pattern recognition, interpretation, and evaluation. In the example this could be the recognition of the other ship, and that this ship is requiring some focus.

- *Projection (Level 3 SA):* This level involves the ability to project the future course of the other ship and how it will interfere with the shift supervisor's ship.

As Endsley (2015) points out these levels can not be seen as a linear process since the situation awareness is continuously updated by going back and forth between the three levels. However,

as a first approach we will, at least for our example, consider the transition between the three levels as a linear process. From a risk analysis perspective it is relevant to model the situation in terms of transitions between the levels. Introduce:

- T_A = Time to perception, i.e., time from the other ship turns into a course conflicting with "our" ship until the shift supervisor pay attention to the other ship.
- T_C = Time to comprehension, i.e., time from the first attention of the other ship is made, until shift supervisor recognizes that this is a ship that might interfere with his ship's course.
- T_P = Time to establish reasonable projections, i.e., the time it takes the shift supervisor to establish a reasonable projection of the approaching ship such that decisions could be made.

7.2 Time to perception modelling

Various approaches could be used for modelling the probability distribution of T_A. In this presentation we will take the perspective that there is a "signal" becoming stronger and stronger. For the example this could be the visibility of the approaching ship in case of fog. The signal is considered as a stochastic process, say $\{S(t), t \geq 0\}$. A range of classes of stochastic processes exist, and in this chapter we only consider the following processes:

- The Wiener process where increments during a short period of time, Δt, are independent and normally distributed with mean value $\mu\Delta t$ and variance $\sigma^2\Delta t$.
- A continuous-time Markov chain. The transition rate between state i and state $i+1$ is given by λ_i.

In the modelling we assume that we could normalize the signal where $0 \leq S(t) \leq 1$.Further it is natural to consider that $T_A \leq t$ even when $S(t) < 1$ which means that the shift supervisor might pay attention to the situation also for weak signal.

In order to determine the probability density function for $T_A \leq t$ we define an awareness function. This function has two interpretations:

1. A static awareness function which is constant throughout the entire period
2. A dynamic awareness function that is "demanded" regularly

Let $g(a)$ be the awareness function where:

- The awareness, A, is a stochastic variable

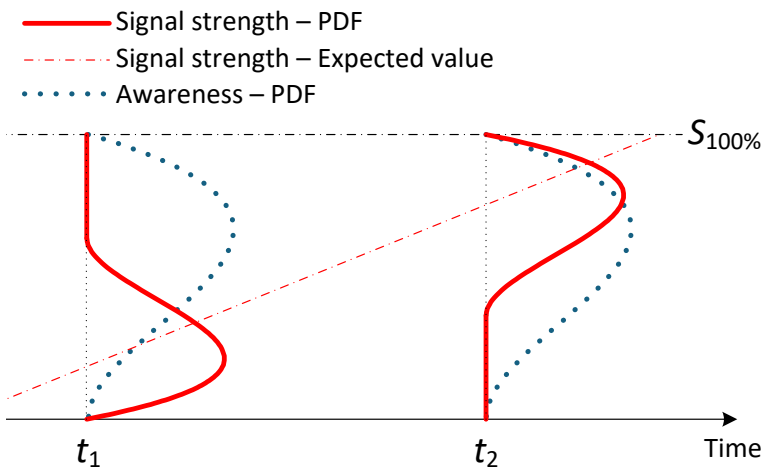


Figure 7.1: Signal strength and awareness as function of time

- $g(a)$ is a probability density function, such that:
- If the awareness is a, the shift supervisor will get attention given that the signal $S(t) > a$
- $g(a)$ is defined on the interval $[0,1]$

Since $g(a)$ is defined on the interval $[0,1]$ it is reasonable to use a beta distribution for the awareness function.

Figure 7.1 depicts the situation. The signal strength is increasing over time, whereas the awareness function, $g(a)$, remains unchanged.

In the situation of a *static* awareness function the arguments are as follows:

- The signal is considered as a stochastic process, say $\{S(t), t \geq 0\}$
- In the actual situation/scenario, the awareness, A, of the shift supervisor takes a fixed value a throughout the scenario. Time to perception, T_A, is given as the lowest value of t such that $S(t) > a$
- To obtain the probability distribution function for T_A we first condition on $A = a$ to obtain the conditional probability distribution function, and then integrate over $g(a)$.

The following paragraphs demonstrate how we could obtain the probability distribution function for T_A given that signal is modelled by a Wiener process with linear drift μ and infinitesimal variance σ_2.

It is well known from the theory of stochastic processes that the time T when the process for the first time reach the level ℓ is inverse-Gauss distributed with parameters $\nu = \ell/\mu$ and $\tau = (\ell/\sigma)^2$.

For the inverse-Gauss distribution, i.e., $X \sim \mathrm{IG}(\nu, \tau)$ we have:

$$f_X(x;\nu,\tau) = \sqrt{\frac{\tau}{2\pi x^3}} \exp\Big(-\frac{\tau(x-\nu)^2}{2\nu^2 x}\Big) \tag{7.1}$$

and

$$F_X(x;\nu,\tau) = \Phi\left(\frac{\sqrt{\tau}}{\nu}\sqrt{x} - \sqrt{\tau}\frac{1}{\sqrt{x}}\right) + \Phi\left(-\frac{\sqrt{\tau}}{\nu}\sqrt{x} - \sqrt{\tau}\frac{1}{\sqrt{x}}\right) e^{2\tau/\nu} \tag{7.2}$$

The expected value and variance are given by:

$$\begin{aligned} \mathrm{E}[X] &= \nu \\ \mathrm{Var}(X) &= \nu^3/\tau \end{aligned}$$

In the Wiener process with parameters μ and σ the time, T, to first passage of the threshold ℓ is then:

$$T \sim \mathrm{IG}(\ell/\mu, (\ell/\sigma)^2)$$

and the expected value and variance are given by:

$$\begin{aligned} \mathrm{E}[T] &= \ell/\mu \\ \mathrm{Var}(T) &= \sigma^2\ell/\mu^3 \end{aligned}$$

In our situation $\ell = a$, i.e., the awareness in the actual situation. This means that given awareness level $A = a$ the conditional probability density function for T_A is given by:

$$f_{T_\mathrm{A}}(t;\nu,\tau|A=a) = \sqrt{\frac{\tau}{2\pi t^3}} \exp\Big(-\frac{\tau(t-\nu)^2}{2\nu^2 t}\Big) \tag{7.3}$$

Note that ν and τ depends on the underlying Wiener process and the value of a:

$$\nu = \nu(a) = a/\mu_\mathrm{S} \tag{7.4}$$

$$\tau = \tau(a) = (a/\sigma_\mathrm{S})^2 \tag{7.5}$$

where subscript S is used to indicate the "signal". The unconditional probability distribution function is given by:

$$f_{T_\mathrm{A}}(t) = \int_0^1 f_{T_\mathrm{A}}(t;\nu,\tau|A=a)g(a)da = \int_0^1 \sqrt{\frac{\tau(a)}{2\pi t^3}} \exp\Big(-\frac{\tau(a)(t-\nu(a))^2}{2\nu(a)^2 t}\Big)g(a)da \tag{7.6}$$

Since $g(a)$ is the probability density function of the awareness, we need to assess the relevant parameters. For the beta distribution with parameters α and β it might be shown that if we know the mean value, say μ_A, and standard deviation, say σ_A, we have:

$$\beta = \left(\frac{\mu_A(1-\mu_A)}{\sigma_A^2} - 1\right)(1-\mu_A) \tag{7.7}$$

$$\alpha = \frac{\mu_A \beta}{1-\mu_A} \tag{7.8}$$

To specify the model we need to give numerical values for the model parameters μ_S, σ_S, μ_A and σ_A (S = signal, A = awareness). These parameters will again depend on the risk influencing factors (RIFs).

When introducing a "static awareness function" we assume that the "signal" increases in magnitude, and as soon as it reaches an "awareness level" the shift supervisor get attention to the situation. The human and organisational RIFs are basically linked to the parameter μ_A.

If we use the "dynamic awareness function" the performance of the shift supervisor is made more explicit. We introduce the following:

- The signal is still considered as a stochastic process, i.e., $\{S(t), t \geq 0\}$
- The shift supervisor is continuously monitoring the situation, that is, he supervises the "normal operation" in addition to the watch keeping required by the watch keeping act. The watch keeping is considered as a Poisson process with some intensity ρ. Watch keeping activities could be scanning the horizon, looking at various instruments at the dashboard etc. Each time such a watch keeping activity takes place, i.e., at time t_i we define the awareness, A_i, such that the first time $S(t_i) \geq A_i$ results in $T_A = t_i$
- To simplify we assume that the A_i's are identical and independently distributed with the probability density function $g(a)$.

The term 'dynamic' is used because the awareness is assumed to change during the scenario in terms of that each time a "watch keeping" take place the awareness will vary. As the signal becomes stronger, the more likely it is that a single "watch keeping" activity will get the attention of the shift supervisor. Finally the "watch keeping" intensity ρ will also influence how fast attention is achieved.

In the dynamic model we separate RIFs that influence ρ and those that influence the awareness in terms of μ_A and σ_A.

It is not possible to find an explicit formula for the probability density function of T_A. A numerical approach is as follows:

- Discretize the signal into $r+1$ states staring in state zero corresponding to $S(0) = 0$ and where state r corresponds to $S(t) = 1$, i.e., $S_{100\%}$ in Figure 7.1

- Establish a transition matrix, **A**, governing the transition process where we assume a Markov process for simplicity
- Use the Markov differential equations to find the time dependent solution in small steps, Δt, i.e., $\mathbf{P}(t+\Delta t) = \mathbf{P}(t)[\mathbf{I}+\mathbf{A}\Delta t]$
- For each integration step calculate the probability that there is an inspection in that interval which "reveals the situation". This "probability" then goes into the probability density function of T_A.
- In the modelling we do not modify the signal process, hence we multiply the "contribution" to the probability density function of T_A with the probability that the situation has not been "revealed". The core Python code is shown below:

```
t = 0
R = 1
x= [0]
y= [0]
Pt = [1,0,0,0,...,0] # The process starts in state 0
IM  = np.eye(N) + np.dot(A,dt) # Integration matrix, i.e., (I + A Delta t)
while t < 2*MTTH: # MTTH = Mean time to "hit", i.e., when S(t) reaches 1
   t += dt
   Pt = np.dot(Pt, IM )
   prob = 0
    for j in range(r+1):
      prob += Pt[j] * bt.cdf(j/r,alpha,beta) # Pr(S(t)=j/r)*Pr(A_i<= j)
   prob *= rho*dt    # rho*dt = Pr(Watch keeping taken place in [t,t+dt])
   x.append(t)          # x-value for plot
   y.append(R*prob*dt) # y-value for plot, R = "survived" up to t without
                                          "attention"
   R *= (1-prob*dt)
```

The x- and y-vectors now contains the probability density function of T_A.

Note the following:

- The transition matrix **A** and the watch keeping rate ρ depend on the risk influencing factors (RIFs). It is required to specify this relation.
- Rather than specifying each transition in **A** it might be easier to specify the expected time the process takes to reach state r and the corresponding standard deviation
  - Let T_r be the time it takes to reach state r, and assume we are able to specify $\mathrm{E}[T_r]$ and $\mathrm{SD}(T_r)$ which depends on the RIFs.

  - Let λ_i be the transition rate from state i to state $i+1$, and let λ_i^{B} be the transition rate to go back from state i to state $i-1$. All other transition rates are consider to be zero.
  - Going "back and forth" in this way corresponds to a Wiener process where the increments could be negative, and thus gives some flexibility in the modelling
  - A simple approach now is to let $\lambda = \lambda_i$ and $\lambda^{\mathrm{B}} = \lambda_i^{\mathrm{B}}$ and then by trial and error find the values of λ and λ^{B} which are consistent with $\mathrm{E}[T_r]$ and $\mathrm{SD}(T_r)$. See Wikipedia Phase-type_distribution for how to calculate moments.

- The models proposed are conceptual models where it is possible to explicitly link the risk influencing factors to the model parameters. However, experiments, use of general results from literature etc. are required to establish the numerical values for model parameters and hyper model parameters.

7.3 Time to comprehension modelling

An important metaphor introduced by Endsley (1995) is the use of schema and pattern matching to schema. This might be a starting point for the time to comprehension modelling. A very simple model is as follows:

- There are n "prototypical" schema in the short term memory of the shift supervisor
- One of these schema is consider to be the "correct schema", i.e., that will enable the shift supervisor to comprehend the situation and get a reasonable understanding of the situation
- All the n schema are considered to be retrieved from the short term memory in an arbitrary order
- Each schema is processed one by one, and processing time is considered to be exponentially distributed with intensity parameter ξ
- The processing of each schema is considered to be independent of previous processing, and the processing goes into a loop described below
- The outcome of processing a schema is as follows: The probability of matching the current situation to a correct (True) schema is $p_{\mathrm{T}} \approx 1$. The probability of matching the current situation to an incorrect (False) schema is $p_{\mathrm{F}} << 1$.
- The first time the correct schema is recognized by the shift supervisor is defining the time to comprehension, i.e., T_{C}

- If an incorrect schema is recognized by the shift supervisor he continues the sense making process with intensity ξ, but now he needs to "escape" from the wrong mental model. The probability of escaping from a wrong mental model is p_E. Whenever escaped from the wrong mental model, the entire search for a correct schema continues.

The model parameters to assign are n, ξ, p_T, p_F and p_E. All of these parameters are considered to depend on risk influencing factors.

To obtain the probability density function for T_C we define the states "search" (searching for the correct schema), "stuck" (stuck with wrong schema) and "success". The process starts in state "search". We integrate the model, and for each time step Δt we apply a transition matrix shown in the Python code below. Note that $\xi \Delta t$ is the probability that a schema matching process terminates in that time interval of length Δt.

```
A = np.eye(3)
A[search][success] = xi * dt * p_T / n
A[search][search] -= xi * dt * p_T / n
A[search][stuck] = xi * dt * (n-1)* p_F / n
A[search][search] -= xi * dt * (n-1)* p_F / n
A[stuck][search] = xi * dt * p_E
A[stuck][stuck] -= xi * dt * p_E
Pt = [1,0,0]
t =0
x=[]
y=[]
prev = 0
while t < T:
    Pt = np.dot(Pt, A )
    x.append(t)          # x-value for plot
    y.append(Pt[success]-prev) # y-value for plot
    prev = Pt[success]
    t += dt
```

7.4 Time to establish reasonable projections and decisions

No explicit model is derived for T_P. If we assume that the significant time factors are the time to get attention to a critical situation, and the time it takes to recognize (comprehend) the situation we may ignore the time it will take to establish a reasonable projection and make a decision, i.e., adjust the course and/or speed. If also effort is required to establish projections and actually act upon this, we need to develop models for T_P. For the time being, we only consider time to attention and time to recognize.

7.5 Human error probability calculations

Given that we have established the probability density functions for time to attention and time to recognition, we can use these to find the probability that the shift supervisor are not able to act upon the situation in due time, i.e., a human error. The following assumptions are made:

- $f_A(t)$ = probability density function for time to attention
- $f_C(t)$ = probability density function for time to comprehend the situation
- Time to attention and time to comprehend are considered to be stochastically independent. This is only valid for a given set of RIF-values. If there is uncertainty in the RIF's, we need to condition on this uncertainty in the modelling. This will not be considered here.
- The time to make projections and make reasonable decisions based on these are not considered
- There is a fixed time limit at disposal, T_L

The probability of failure is now the probability that the shift supervisor is not able to comprehend the situation adequately within the time limit at disposal, i.e.,:

$$\text{HEP} = \int_0^\infty f_A(t)(1 - F_C(T_L - t)\,dt \tag{7.9}$$

here $F_C(t)$ is the cumulative distribution function for time to comprehend.

If the time limit at disposal, T_L, is a stochastic variable with probability distribution function $f_L(t)$ we use:

$$\text{HEP} = \int_{u=0}^\infty f_L(u) \int_{t=u}^\infty f_A(t)(1 - F_C(u - t)\,dt\,du \tag{7.10}$$

Appendix A

Risk assessment & NS5814

A.1 Introduction

NS 5814 Requirements for risk assessments is a Norwegian standard for carrying out risk assessments. The standard has come in several editions, and the latest edition is from 2021.

NS 5814 is a standard for risk assessments that must cover a broad scope, i.e., cover different disciplines and industries. It also means that one encounters slightly different terminology. However, despite different terminology, the logic of risk assessments is essentially the same, and the differences should not stand in the way of a common understanding of what a risk assessment is.

A.2 Terminology

Some important definitions from NS 5814 are presented. It should be emphasized that these definitions slightly deviates from some of the definitions given in other textbooks and this course compendium.

☞ **Object of analysis**: Physical or organisational system, device, phenomenon or activity covered by the risk assessment

☞ **Barrier**: Measures intended to influence the course of events so that the incident does not occur or have undesirable consequences

☞ **Hazard**: Conditions that may lead to an adverse event

☞ **Consequence**: Loss of value as a result of an undesirable event

☞ **Risk**: Uncertainty related to whether an unwanted event will occur and what consequences it may have

☞ **Risk analysis**: Systematic approach to describing risk

☞ **Risk evaluation**: Process for assessing whether safety objectives have been achieved by comparing the results of the risk analysis with the evaluation criteria, and providing the decision-maker with a recommendation on risk management

☞ **Risk assessment**: Overall process consisting of establishing a framework for the risk assessment, identifying undesirable incidents, risk analysis and risk evaluation

☞ **Safety objectives**: Established goals for safeguarding of values

☞ **Evaluation criteria**: Explicit criteria used to verify the safety objectives

A.3 Steps

Figure A.1 illustrates the steps for conducting the risk assessment proposed by NS 5814. The steps are discussed in the following.

A.3.1 Framework for the risk assessment

This includes several sub-steps. Under “Purpose, requirements and delimitations”, it is particularly important to specify which decisions the risk assessment is to support. For example, whether safety goals can be achieved with chosen solutions, which risk-reducing measures are most effective, etc.

Under “Values to be protected”, you will typically identify people, equipment, critical functions, ability to produce, etc. Identified values will later form the basis for the safety objectives.

Establishing safety objectives and evaluation criteria for risk is crucial for the results of the risk analysis to support the decision-making situations that have been established. In earlier versions of NS 5814 and in other presentations of risk assessments, this step is described as the determination of *risk acceptance criteria*. However, the latest version of NS 5814 does not use the term ‘ risk acceptance criteria’.

The approach is to first establish overarching security requirements related to the assets to be protected. It can be result goals, functional requirements, technical requirements or re-

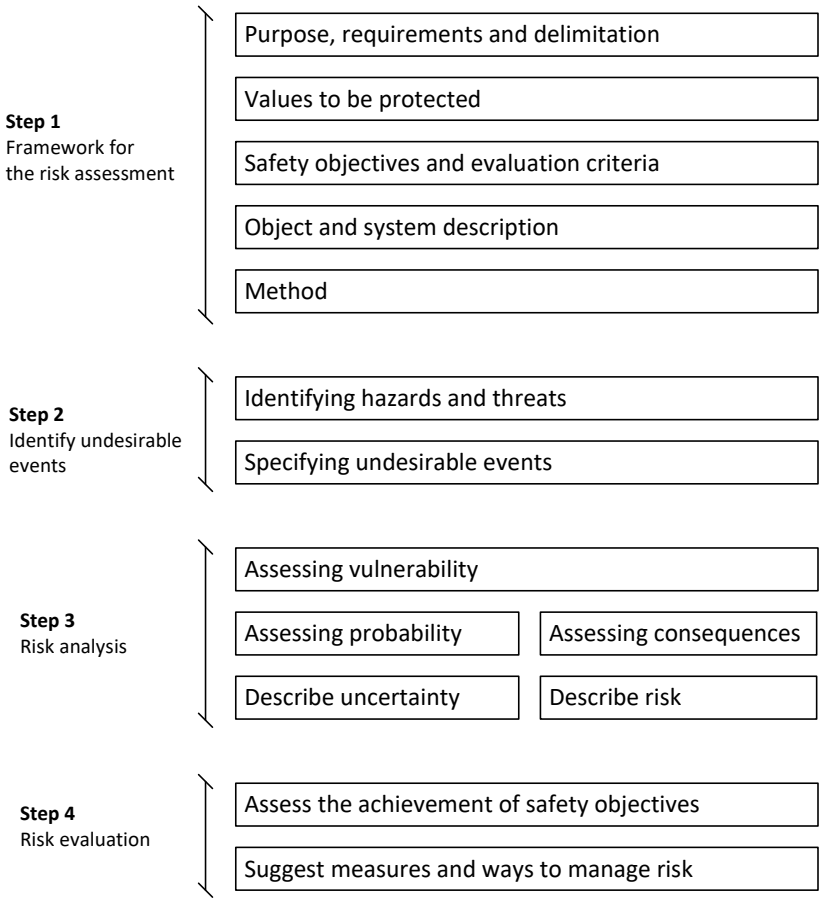


Figure A.1: Steps for conducting the risk assessment proposed by NS 5814

quirements to optimize solutions. The safety objectives will then be operationalised and broken down into specific sub-goals or evaluation criteria against which the results of the risk analysis can be compared. Examples of evaluation criteria can be:

- Performance requirements for barriers
- Target-borne indicators of risk, e.g., related to individual risk or major accident risk
- Documentable redundancy
- That a solution is at least as good as a pre-accepted solution

A.3.2 identify undesirable events/Hazardous events

This step consists of first identifying hazards and threats, and then specifying associated undesirable events. A number of techniques exist to support this step, depending on the type of risk analysis to be carried out. A separate lesson deals with identifying and structuring hazards.

An example of hazard can be that you work at heights, and an associated undesirable event is that you fall down. Note that hazards and threats are conditions that exist more or less all the time, while undesirable events are when the hazard or threat is triggered.

Bow-tie model Identified undesirable events must be described as specifically as possible. It is often appropriate to present a sequence of events, or a so-called accident scenario. A bow-tie model is an appropriate way to describe the causes that lead to the undesirable event, and possible consequences if the undesirable event occurs.

In the risk analysis itself, we often use such a bow-tie model as a starting point when assessing the probability of the undesirable event occurring, and the consequences given that the undesirable event has occurred. Figure A.2 outlines the content of a bow-tie model.

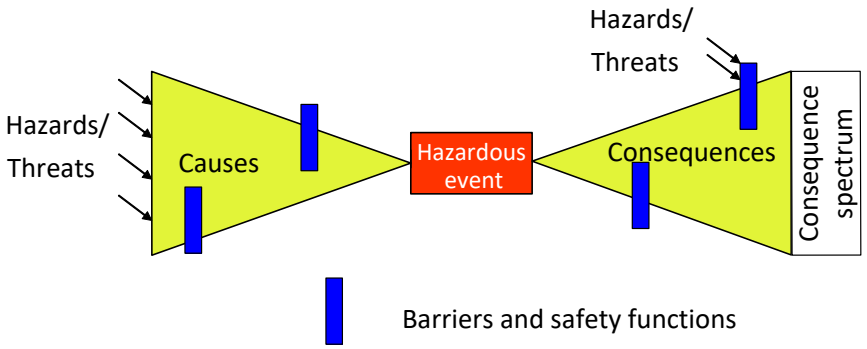


Figure A.2: Bow-tie model

A.3.3 Risk analysis

The risk analysis includes four main elements:

Assessing vulnerability

Here, one must assess how vulnerable the object of analysis is to undesirable incidents occurring and having undesirable consequences. The vulnerability of the object of analysis is mapped by studying how the course of events may develop. By vulnerability, we mean the ability of the object of analysis to resist an unwanted event and preserve or resume its function after the event has occurred.

Assessing probability

The probability that the undesirable incident will occur shall be determined. Note that the statement is a subjective opinion based on our knowledge of the object of analysis, and not

an estimate of a true value. This applies whether the probability is determined quantitatively or semi-quantifiably (high/low).

Note that there is always uncertainty associated with whether an event will occur, and this uncertainty can then be expressed through probability.

There are many analysis techniques for assessing probability. The most commonly used technique is fault tree analysis. There is a separate lesson that deals with the fault tree analysis.

Assessing consequences

An assessment must be made of the consequences that the undesirable incident may have for the values described earlier. Consequences for different values are described separately and categorised according to severity. The overall consequences are a result of both the undesirable event and the consequential events (see the loop model). The effect of barriers to consequential events and consequences must be included in the assessment.

Consequences can be described specifically through, for example, number, scope and duration. More abstract consequences, such as loss of trust and security, can be assessed in terms of the degree of loss.

The most common technique for analysing the consequences of adverse events is incident tree analysis. A separate lesson covers this technique.

Describe uncertainty

The strengths and weaknesses of the knowledge base for all parts of the risk assessment will be assessed. The knowledge base is described together in connection with the results of the risk assessment. Any weaknesses in the knowledge base must be identified in the description of risk and taken into account in recommendations and decisions related to risk management that follows.

Great uncertainty due to lack of knowledge may indicate a precautionary principle when making decisions; that the decision-maker takes into account that the risk may be higher than stated in the assessment. Similarly, a strong knowledge base may indicate that the indication of risk should be given great weight. Some indicators to assess the strength of the evidence base are:

- degree of understanding of the event, the object of analysis, and the system;
- the relevance of knowledge, experience and research in the field;
- the scope of data and the involvement of professional knowledge.

A.3.4 Describe risk

The results of the risk analysis shall be described so that they can be evaluated in accordance with established safety objectives and evaluation criteria defined initially. It should be clear that risk is a complex entity that contains a lot of information.

Risk can be described by stating the probability of undesirable incidents and the consequences they may have, how vulnerability affects the probability and consequences, and what contributes to uncertainty. Trends that may change the risk outlook in the future may be included in the description.

A.3.5 Risk evaluation

It shall be assessed and described to what extent established safety requirements have been achieved by comparing the results of the risk analysis with the evaluation criteria that have been defined. The evaluation will take a position on what the analysis results say about risk.

The evaluation must provide answers to one or more of the following points:

- To what extent is there a correlation between the assessed risk and the safety objectives? The discussion is based on the evaluation criteria.
- Which solution involves the lowest risk if there are alternative solutions? Highlight the factors that distinguish the risks of different alternatives and how the alternatives are ranked with regard to risk.
- Has the risk been sufficiently elucidated to assess whether the safety objectives have been achieved? Is the knowledge base good enough? Has the process been good enough? Are the decision-maker's information needs met?
- Which aspects of the object of analysis contribute most to risk? Describe any special features of the analysis object.

A recommendation must also be made on how the decision-maker should follow up the risk assessment based on the evaluation. Any risk-reducing measures must be related to the identified risk factors in the analysis, so that the expected risk-reducing effect can be assessed. Distinguishing between probability-reducing and consequence-reducing measures can help explain the effect. The loop model is a tool for illustrating measures.

Measures are assessed based on the ALARP principle, the precautionary principle, expected risk-reducing effect, cost-effectiveness, cost-benefit or other assessment criteria.

The ALARP principle stands for "As Low As Reasonably Practicable" and means that all risk-reducing measures must be implemented unless they have disproportionately large costs or disadvantages.

A.3.6 Uncertainty

Uncertainty is a fundamental concept in risk assessments. NS5814 links risk to undesirable incidents with subsequent consequences for values. There is uncertainty related to whether the incidents will occur and what the consequences will be. When we prepare a risk assessment, we cannot know which incidents will occur and what losses may result from these.

In some representations, a distinction is made between lack of knowledge (epistemic uncertainty) and/or random variation (aleatoric uncertainty or variability).

Examples:

- Epistemic uncertainty (lack of knowledge, more knowledge may reduce uncertainty):
  - What is the average temperature in January around 2050?
  - Is the rock structure such that there will be a lot of water penetration?
- Aleatoric uncertainty (variability, more knowledge does not help):
  - What is the average temperature in January next year?
  - How many eyes will there be in the next roll of the dice?

With such a distinction, one can also characterize, for example, uncertainty in model parameters to belong to epistemic uncertainty. Given the value of the model parameters, there will still be aleatory uncertainty, i.e. "from time to time variation".

NS5814 does not distinguish between epistemic and aleatory uncertainty. The approach in NS5814 is to study uncertainty in the relation to time axis:

- Present
- Past
- Future

The following are some questions for each of these phases that the risk assessment should highlight. NS5814 also gives specific advice on how to handle the uncertainty, which is not reproduced here.

Uncertainty related to framework conditions (present)

- Has the analysis group understood which decisions the risk assessment is supposed to support?
- Is it conceivable that other framework conditions for time, finances or other resources could have had a significant impact on the result?
- Has the analysis group chosen a process and methods that support the goal of the risk assessment?
- Does the analysis group have sufficient overview and knowledge of the system and the object of analysis, including hazards and threats?
- To what extent was the analysis group prepared?
- To what extent is the analysis based on the best available knowledge in the area?
- Do the models used in the risk assessment provide a sufficient understanding of systems and phenomena, or have we made simplifications that affect the results to a large extent? To what extent do the models used in the analysis reflect real-world relationships?

Uncertainties related to data and information (past)

- To what extent is the analysis based on data and information from previous events?
- To what extent is the background and context of data and information known?
- To what extent do the quantitative data represent real quantities in the entire population?
- To what extent does the analysis group have the correct knowledge of historical events?

Uncertainties related to the future

- Does the risk assessment provide a good description of which incidents may occur and their possible consequences?
- To what extent does the risk assessment challenge the understanding of possible futures and developments?
- Have all assumptions and assumptions for the risk assessment been identified and described?
- Has the significance of the most important assumptions and assumptions been assessed?

Appendix B

Fault tree analysis

B.1 Introduction

A fault tree is a logic diagram that displays the relationships between a potential critical event (accident) in a system and the reasons for this event. The reasons may be environmental conditions, human errors, normal events (events which are expected to occur during the life span of the system) and specific component failures. A properly constructed fault tree provides a good illustration of the various combinations of failures and other events which can lead to a specified critical event. The fault tree is easy to explain to engineers without prior experience of fault tree analysis.

An advantage with a fault tree analysis is that the analyst is forced to understand the failure possibilities of the system, to a detailed level. A lot of system weaknesses may thus be revealed and corrected during the fault tree construction.

A fault tree is a *static* picture of the combinations of failures and events which can cause the TOP event to occur. Fault tree analysis is thus not a suitable technique for analysing dynamic systems, like switching systems, phased mission systems and systems subject to complex maintenance strategies.

A fault tree analysis may be qualitative, quantitative or both, depending on the objectives of the analysis. Possible results from the analysis may e.g. be:

1. A listing of the possible combinations of environmental factors, human errors, normal events and component failures that can result in a critical event in the system.

2. The probability that the critical event will occur during a specified time interval.

Figure B.1 shows an example fault tree for the bike.
The analysis of a system by the fault tree technique is normally carried out in five steps:

1. Definition of the problem and the boundary conditions.

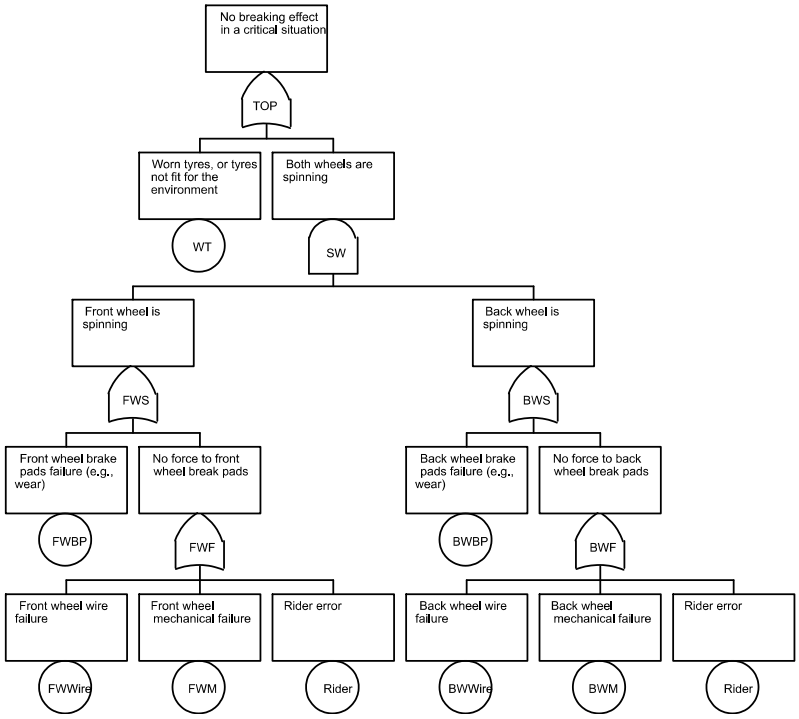


Figure B.1: FTA example for a bike

2. Construction of the fault tree.

3. Identification of minimal cut and/or path sets.

4. Qualitative analysis of the fault tree.

5. Quantitative analysis of the fault tree.

In the following we will present the basic elements of standard fault tree analysis. Then we will conclude this chapter by presenting a numerical example illustrating how the technique could be utilised in relation to maintenance optimisation.

B.2 Fault tree construction

B.2.1 Fault tree diagram, symbols and logic

A fault tree is a logic diagram that displays the connections between a potential system failure (TOP event) and the reasons for this event. The reasons (Basic events) may be environmental conditions, human errors, normal events and component failures. The graphical symbols used to illustrate these connections are called “logic gates”. The output from a logic gate is determined by the input events.

The graphical layout of the fault tree symbols are dependent on what standard we choose to follow.

B.2.2 Definition of the Problem and the Boundary Conditions

This activity consists of:

1. Definition of the critical event (the accident) to be analysed.
2. Definition of the boundary conditions for the analysis.

The critical event (accident) to be analysed is normally called the TOP event. It is very important that the TOP event is given a clear and unambiguous definition. If not, the analysis will often be of limited value. As an example, the event description "Fire in the plant" is far too general and vague. The description of the TOP event should always answer the questions: **What, where** and **when**?

What: Describes what type of critical event (accident) is occurring, e.g., collision between two trains.

Where: Describes where the critical event occurs, e.g., on a single track section.

When: Describes when the critical event occurs, e.g., during normal operation.

A more precise TOP event description is thus: "Collision between two trains on a single track section during normal operation".

1. To get a consistent analysis, it is important that the *boundary conditions* for the analysis are carefully defined. By boundary conditions we mean: **The physical boundaries of the system**. What parts of the system are to be included in the analysis, and what parts are not?
2. **The initial conditions**. What is the operational state of the system when the TOP event is occurring? Is the system running on full/reduced capacity? Which valves are open/closed, which pumps are functioning etc.?
3. **Boundary conditions with respect to external stresses**. What type of external stresses should be included in the analysis? By external stresses we here mean stresses from war, sabotage, earthquake, lightning etc.
4. **The level of resolution**. How far down in detail should we go to identify potential reasons for a failed state? Should we as an example be satisfied when we have identified the reason to be a "valve failure", or should we break it further down to failures in the valve housing, valve stem, actuator etc.? When determining the required level of resolution, we should remember that the detail in the fault tree should be comparable to the detail of the information available

B.2.3 Construction of the Fault Tree

The fault tree construction always starts with the TOP event. We must thereafter carefully try to identify all fault events which are the immediate, necessary and sufficient causes that result in the TOP event. These causes are connected to the TOP event via a logic gate. It is important that the first level of causes under the TOP event is developed in a structured way. This first level is often referred to as the TOP structure of the fault tree. The TOP structure causes are often taken to be failures in the prime modules of the system, or in the prime functions of the system. We then proceed, level by level, until all fault events have been developed to the required level of resolution. The analysis is in other words deductive and is carried out by repeated asking "What are the reasons for...?"

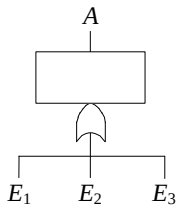


Figure B.2: OR-gate

Figure B.2 shows the OR-gate indicating that the output event A occurs if any of the input events E_i occurs. In relation to the bike example with TOP event "No breaking effects" the two events: "No friction" and "both wheels spinning" are connected by an OR gate since any of these events will lead to the TOP event.

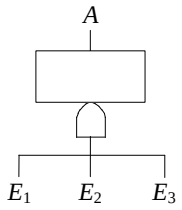


Figure B.3: AND-gate

Figure B.3 shows the AND-gate indicating that the output event A occurs only when all the input events E_i occurs simultaneously. In the bike example, "Front wheel is spinning" and "Rear wheel is spinning" are connected by an AND gate, since both these event have to occur in order to full fill the requirement that both wheels are spinning.

Figure B.3 shows the Basic event representing a basic equipment fault or failure that requires no further development into more basic faults or failures. An example of a basic event in the bike example is "Breakage in break wire".

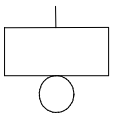


Figure B.4: BASIC-event

B.3 Identification of Minimal Cut- and Path Sets

A fault tree provides valuable information about possible combinations of fault events which can result in a critical failure (TOP event) of the system. Such a combination of fault events is called a cut set.

*A**cut set** in a fault tree is a set of Basic events whose (simultaneous) occurrence ensures that the TOP event occurs. A cut set is said to be **minimal** if the set cannot be reduced without loosing its status as a cut set.*

*A**path set** in a fault tree is a set of Basic events whose non-occurrence (simultaneously) ensures that the TOP event does not occur. A path set is said to be **minimal** if the set cannot be reduced without loosing its status as a path set.*

In practice only minimal cut sets are used for evaluation of fault trees. To find the minimal cut sets we apply the MOCUS algorithm (Method Of obtaining Cut Sets). The MOCUS algorithm essentially contains the following elements:

1. Start with the TOP event
2. As the algorithm proceeds, the result is stored in a matrix like format of rows and columns
3. AND- and OR-gates are resolved by replacing the gate with it's "children" in the fault tree diagram
4. An AND-gate means that the gate is replaced by new elements for the row(s) it is found
5. An OR-gate means that the gate is replaced by as many rows that the gate has children, where each child is inserted at the position of the OR-gate being replaced
6. When all gates are replaced, we remain with only the basic events, where each row corresponds to a cut set

Note that the the cut sets will not necessarily be be minimal. To make the cut sets minimal we have to:

1. Replace duplicates of one event with only one occurrence of that event in each row
2. If one row is a sub set of another row, then the larger of these two rows (representing non-minimal cut sets) is removed

The MOCUS algorithm is demonstrated here: http://folk.ntnu.no/jvatn/eLearning/TPK4120/Examples/MOCUS.html in relation to the example used in the lectures.

B.3.1 *k*oo*n* gate

The *k*oo*n* gate is something "between" the AND and OR gate. A *k*oo*n* gate occurs if *k* out of the *n* inputs occur. Note that in FTA we focus on fault states, i.e., an event occurring means a failure, hence the "voting" in FTA is different from in RBD. To clarify, the following notation is often used:

- $k\text{oo}n : G$ is used if we consider the functioning of components (G=Good). The system (block) functions if *k* or more out of the *n* components are functioning
- $k\text{oo}n : F$ is used if we consider the fault of components (F=Fault state). The system (gate/-TOP event) occurs if *k* or more out of the *n* inputs are occurring (i.e., in a fault state)

Note the following relation:

$$k\text{oo}n : G = (n–k+1)\text{oo}n : F$$
$$k\text{oo}n : F = (n–k+1)\text{oo}n : G$$

Consider a system with three pumps each having 50% capacity. The system functions if at least 2 of the pumps are functioning. In an RBD we then use the $2\text{oo}3 : G$ block for this system, and for the FTA we use the $k\text{oo}n : F = n–k+1\text{oo}n : G = 3-2+1\text{oo}3 = 2\text{oo}3$ gate.

If we have 4 such pumps, the RBD representation is $2\text{oo}4 : G$, and in FTA we use the $k\text{oo}n : F = n–k+1\text{oo}n : G = 4-2+1\text{oo}4 = 3\text{oo}4$ gate meaning that 3 or more pumps must be in a fault state in order to give a system failure (TOP event).

Computerized FTA programs will offer the $k\text{oo}n : F$ as part of the drawing palette. For manual construction of a fault tree with a $k\text{oo}n : F$ gate we can use an OR-gate followed by several AND-gates. Each AND-gate is then a sub-set with *k* out of the *n* inputs. There are altogether $\binom{n}{k}$ ways we may choose *k* inputs out of *n* inputs, hence we will have $\binom{n}{k}$ AND-gates to put under the OR-gate.

B.4 Qualitative Evaluation of the Fault Tree

A qualitative evaluation of the fault tree may be carried out on the basis of the minimal cut sets. The importance of a cut set depends obviously on the number of Basic events in the cut set. The number of different Basic events in a minimal cut set is called the *order* of the cut set. A cut set of order one is usually more critical than a cut set of order two, or higher. When we have a cut

set with only one Basic event, the TOP event will occur as soon as this Basic event occurs. When a cut set has two Basic events, both of these have to occur at the same time to cause the TOP event to occur.

Another important factor is the type of Basic events in a minimal cut set. We may rank the criticality of the various cut sets according to the following ranking of the Basic events:

1. Human error
2. Failure of active equipment
3. Failure of passive equipment

The ranking is based on the assumption that human errors occur more frequently than active equipment failures, and that active equipment is more failure-prone than passive equipment (an active or running pump is for example more exposed to failures than a passive standby pump).

B.5 Quantitative analysis

In the quantitative part of a fault tree analysis the main objective is to calculate the following metrics:

- $Q_0(t)$ = Probability that the TOP-event occurs at time t
- $F_0(t)$ = Expected number of TOP-event occurrence per unit time at time t
- $I(i \mid t)$ = Importance metric for basic event i at time t

For the calculations we need the minimal cut set as well as basic event frequencies and probabilities.

B.5.1 Upper Bound Approximation, $Q_0(t)$

Assume that we have found the minimal cut sets of the fault tree, i.e., K_j. Further assume that the minimal cut sets do not contain common components, hence they are independent (also provided that the components are independent). We may now arrange the cut set in a series structure as indicated in Figure B.5: Let E_j denote the event that cut set number j is occurring. The probability that cut set number j is occurring is found by:

$$\Pr(E_j) = \check{Q}_j(t) = \prod_{i \in K_j} q_i(t)$$

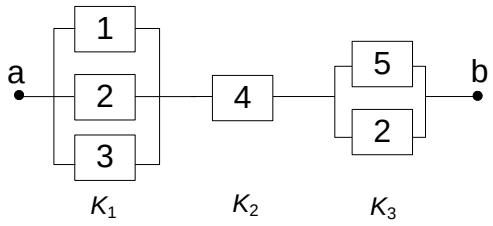


Figure B.5: Example cut set structure

We now have

$$\begin{aligned} Q_0(t) = \Pr(\text{TOP event occurs at time } t) &= 1 - \Pr(\text{TOP event does not occur at time } t) \\ &= 1 - \Pr(\text{No cut set occurs at time } t) \end{aligned}$$

Since the cut sets are independent, and the probability that cut set number j is occurring is given by $\check{Q}_j(t)$, we have:

$$Q_0(t) = 1 - \prod_{j=1}^{k} (1 - \check{Q}_j(t))$$

where

$$\check{Q}_j(t) = \prod_{i \in K_j} q_i(t)$$

Generally there might be some basic events that occur in two or more cut sets, hence the cut sets are *dependent*, and it may be proven that the formula represents an upper bound for the TOP event probability:

$$Q_0(t) \leq 1 - \prod_{j=1}^{k} (1 - \check{Q}_j(t))$$

Hence, we may use:

$$Q_0(t) \approx 1 - \prod_{j=1}^{k} (1 - \check{Q}_j(t))$$

which is referred to as the upper bound approximation and is usually considered to be a good approximation when the $q_i(t)$s are small.

To argue for the less or equal sign we realize that cut sets are “positive dependent” if they

have common components. For two cut sets we have

$$\Pr(E_1^C \cap E_2^C) = \Pr(E_1^C | E_2^C) \Pr(E_2^C) > \Pr(E_1^C) \Pr(E_2^C)$$

and

$$Q_0 = 1 - \Pr(E_1^C \cap E_2^C) < 1 - \Pr(E_1^C) \Pr(E_2^C) = 1 - (1 - \check{Q}_1)(1 - \check{Q}_2)$$

and we may give similar arguments for more two or more cut sets.

B.5.2 The Inclusion-Exclusion Principle, $Q_0(t)$

Referring to Figure B.5 it is also obvious that we may write:

$$Q_0(t) = \Pr(\cup_j E_j)$$

A challenge here is to find the probability of the union of events. For two events A and B we have $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$. For more than two events (cut sets) this becomes more complicated, and we have to use the general addition theorem in probability:

$$Q_0(t) = \Pr(\cup_j E_j) = \sum_j \Pr(E_j) - \sum_{i<j} \Pr(E_i \cap E_j) + \sum_{i<j<k} \Pr(E_i \cap E_j \cap E_k) - \ldots$$

To find $\Pr(E_i \cap E_j)$, $\Pr(E_i \cap E_j \cap E_k)$ is straight forward since these intersections of events are in fact intersection of a set of basic events, and we may multiply the corresponding probabilities as we have done for a single minimal cut set. The challenge is the number of terms we have to calculate. As a starting point we can only take the first sum, i.e., adding the cut set occurrences for each cut set. A slightly better approach would be to subtract the next sum. There are some ways we can optimize the calculations, and finding bounds for the answer to use as a stopping rule, see the textbook. Very often the inclusion-exclusion principle is used by only adding the cut set probabilities:

$$Q_0(t) \approx \sum_{j=1}^{k} \check{Q}_j(t) \tag{B.1}$$

which is faster than the upper bound approximation, but less accurate.

The next challenge is to find the basic event probabilities, $q_i(t)$. Three situations are often considered:

B.5.3 Non-repairable components

If a component cannot be repaired, the probability that it is in a fault state at time t equals $1 - R(t)$, and provided that the component has an exponentially distributed life time, we therefore have:

$$q_i(t) = 1 - e^{-\lambda_i t} \tag{B.2}$$

where λ_i is the constant failure rate of the component.

B.5.4 Repairable components

To derive $q_i(t)$ for a repairable components we may use Markov analysis. The probability that the component is in a fault state at time t is then shown to be (according to eq. 8.22):

$$q_i(t) = \frac{\lambda_i}{\mu_i + \lambda_i}\left(1 - e^{-(\lambda_i+\mu_i)t}\right) \tag{B.3}$$

where λ_i is the constant failure rate of the component, and $\mu_i = 1/\mathrm{MDT}_i$ is the constant repair rate. When t is large compared to $\frac{1}{\lambda_i+\mu_i}$ we have

$$q_i(t) \approx \frac{\lambda_i}{\mu_i + \lambda_i} \approx \lambda_i \mathrm{MDT}_i \tag{B.4}$$

if repair times are short compared to failure times. If this holds, it is safe to use this approximation when $t > 3\mathrm{MDT}_i$, where MDT_i is the mean time to restoration for the component.

B.5.5 Periodically tested components

For components with a hidden function, it is usual to perform a functional test at fixed time intervals, say τ_i, to verify that the component is able to carry out it's function. Imay be shown that the (on demand) failure probability of such a component is given by:

$$q_i(t) \approx \lambda_i \tau_i / 2 \tag{B.5}$$

q_i is often referred to as the probability of failure on demand (PFD).

B.5.6 TOP event frequency, $F_0(t)$

$F_0(t)$ denotes the expected number of occurrences of the TOP event per unit time. In principle we may calculate $F_0(t)$ at various point of times, but usually we focus on the steady state situation, and therefore we omit the time dependency, i.e., we seek F_0.

The arguments are as follows:

- We know the minimal cut sets
- If one cut set should be the "contributor" to the TOP event to occur, the other cut sets cannot be occurring
- For a basic event in one cut set to bring the cut set to occur, requires that all other basic events in that cut set are occurring

Let $C_{\mathcal{K}}$ denote a minimal cut set, then the cut set occurrence frequency is given by:

$$\breve{w}_{\mathcal{K}} = \sum_{i \in C_{\mathcal{K}}} w_i \prod_{\ell \in C_{\mathcal{K}}, \ell \neq i} q_\ell \tag{B.6}$$

where w_i is the ROCOF of basic event i, and q_l is the probability that basic event l is occurring.

The ROCOF is the rate of occurrence of failures. To define the ROCOF we need to have a stochastic process perspective, i.e., we consider what is happening in a time interval rather when things are happening in this interval. Let $N(t)$ be the number of failures that occur in $(0, t]$ and let $W(t) = \mathrm{E}[N(t)]$. The ROCOF at time t is now defined by

$$w(t) = \lim_{\Delta t \to 0} \frac{\mathrm{E}[N(t+\Delta t) - N(t)]}{\Delta t} = \lim_{\Delta t \to 0} \frac{W(t+\Delta t) - W(t)}{\Delta t} = \frac{d}{dt} W(t) \tag{B.7}$$

To obtain the TOP event frequency we may now sum over the $\breve{w}_{\mathcal{K}}$'s. However, note that $\breve{w}_{\mathcal{K}}$ will not contribute to the TOP event frequency if one of the other cut set is already in a fault state, hence the TOP event frequency is better approximated by:

$$F_0 = w_{\mathrm{TOP}} \approx \sum_{\mathcal{K}=1}^{k} \breve{w}_{\mathcal{K}} \prod_{j=1, j \neq \mathcal{K}}^{k} (1 - \breve{Q}_j) \approx \sum_{\mathcal{K}=1}^{k} \breve{w}_{\mathcal{K}} \frac{1 - Q_0}{1 - \breve{Q}_{\mathcal{K}}} \tag{B.8}$$

The formula in Equation (B.8) is the best we can do, but usually $\breve{Q}_j$ is rather small, and it will be sufficient to use

$$F_0 \approx \sum_{\mathcal{K}=1}^{k} \breve{w}_{\mathcal{K}} = \sum_{\mathcal{K}=1}^{k} \sum_{i \in C_{\mathcal{K}}} w_i \prod_{\ell \in C_{\mathcal{K}}, \ell \neq i} q_\ell \tag{B.9}$$

The ROCOF of the basic events is usually found by the failure rate, say λ_i. However, a more exact calculation will also take into account the downtime on basic event level, i.e., we may use:

$$w_i = \lambda_i (1 - q_i) \approx \lambda_i \tag{B.10}$$

B.6 Reliability Importance Metrics

In the literature very many reliability importance metrics are presented. We only focus on the following:

- Birnbaum's metric
- Improvement Potential
- The criticality importance metric
- Fussel-Vesley's metric

In principle a metric is linked to basic events. Very often these basic events are component failures, hence the term component importance is often used. There are many reasons to investigate component importance:

- Considering improving the inherent reliability of critical components
- Establish a preventive maintenance program for the most critical components
- Ensure that we have sufficient spare parts for critical components
- Considering implementing (extra) redundancy at component level for the most critical components
- Given that we have a system failure, which component is the most likely to have caused this?

Several measures are discussed, and the various measures will have their strength and weakness to answer the questions above.

B.6.1 Birnbaum's Metric of Reliability Importance

Birnbaum's metric of reliability importance of a component is a sensitivity measure expressing the change in system reliability if component i is slightly changed, i.e.,;

$$I^{\mathrm{B}}(i \mid t) = \frac{\partial Q_0(t)}{\partial q_i(t)} \tag{B.11}$$

It follows that a small change $\Delta p_i(t)$ in the component reliability will result in the following change in system reliability:

$$\Delta Q_0(t) = I^{\mathrm{B}}(i \mid t)\Delta q_i(t) \tag{B.12}$$

A disadvantage with Birnbaum's metric is that it is difficult to calculate. If we are able to write down the system reliability function, it should be rather easy to find Birnbaum's measure. But in practice we will not be able to write down the TOP event probability, and hence we cannot derive Birnbaum's metric. In some cases we may utilize that:

$$I^{\mathrm{B}}(i \mid t) = Q_0(t \mid q_i = 1) - Q_0(t \mid q_i = 0)$$

It may be shown that $I^{\mathrm{B}}(i \mid t)$ is the probability that component i is critical at time t. This is a valuable result used in maintenance optimization. Often we need to calculate the expected cost of a failure of a specific component. The contribution to downtime depends on whether the system is down or not, and if a failure will cause a system failure. The Birnbaum's metric is exactly what we need, i.e., we should only include downtime cost if the component under consideration is critical, and $I^{\mathrm{B}}(i \mid t)$ is then used for calculating this probability.

B.6.2 Improvement Potential

The Iimprovement Potential states how much the system reliability will increase if component i is replaced with a perfect component:

$$I^{\mathrm{IP}}(i \mid t) = Q_0(t) - Q_0(t \mid q_i = 0) \tag{B.13}$$

It is easy to show the following relation to Birnbaum's metric:

$$I^{\mathrm{IP}}(i \mid t) = I^{\mathrm{B}}(i \mid t) q_i(t) \tag{B.14}$$

B.6.3 Criticality Importance

The criticality importance metric $I^{\mathrm{CR}}(i \mid t)$ of component i at time t is the probability that component i is critical for the system and is failed at time t, when we know that the system is failed at time t. It is easy to show the following relation to Birnbaum's metric:

$$I^{\mathrm{CR}}(i \mid t) = \frac{I^{\mathrm{B}}(i \mid t) \cdot q_i(t)}{Q_0(t)}$$

Fussell-Vesely's Metric

The Fussell-Vesely's importance metric $I^{\mathrm{FV}}(i \mid t)$ of component i at time t is the probability that at least one minimal cut set that contains component i is failed at time t, when we know that the system is failed at time t.

In order to calculate $I^{\mathrm{VF}}(i \mid t)$ we need some reasoning. We simplify and skip the index t. Now introduce the following notation (we use the terminology "component" whereas the precise word would be "basic event"):

- D_i: At least one minimal cut containing component i is failed
- C: The system is failed
- m_i: Number of minimal cut set containing component i
- E_j^i: Minimal cut set j containing component i is failed

From the definition we have:

$$I^{\mathrm{FV}}(i) = \Pr(D_i \mid C) = \frac{\Pr(D_i \cap C)}{\Pr(C)} \tag{B.15}$$

Since D_i is a subset of C, then $D_i \cap C = D_i$ and we have:

$$I^{\mathrm{FV}}(i) = \frac{\Pr(D_i)}{\Pr(C)} \tag{B.16}$$

To find $\Pr(D_i)$ we use the same approach as for the "upper bound' approximation for Q_0. However, note that $D_i = E_1^i \cup E_2^i \cup \cdots \cup E_{m_i}^i$ where the union is only taken over minimal cut sets containing component i. This gives:

$$\Pr(D_i) = 1 - \Pr({E_1^i}^C \cap {E_2^i}^C \cap \cdots \cap {E_{m_i}^i}^C) \leq 1 - \Pr({E_1^i}^C)\Pr({E_2^i}^C)\cdots\Pr({E_{m_i}^i}^C)$$

$\Pr({E_j^i}^C)$ is then obtained by one minus the probability for the event that minimal cut set j is failed, i.e., $\Pr({E_j^i}^C) = 1 - \check{Q}_j^i = 1 - \prod_{l \in K_j} q_l$. The following approximation is usually sufficient to calculate Fussell-Vesely's measure:

$$I^{\mathrm{FV}}(i) \approx \frac{1 - \prod_j^{m_i}(1 - \check{Q}_j^i)}{Q_0}$$

where the product is over minimal cut sets which contain component i.

If cut set failure probabilities are small, a faster approximation is given by:

$$I^{\mathrm{FV}}(i) \approx \frac{\sum_j^{m_i} \check{Q}_j^i}{Q_0} \tag{B.17}$$

where the sum is over minimal cut sets which contain component i.

By comparing the definition of $I^{\mathrm{CR}}(i)$ and $I^{\mathrm{FV}}(i)$, we see that these measures are rahter close to each other. Thus by assuming $I^{\mathrm{CR}}(i) \approx I^{\mathrm{FV}}(i)$, we could easily get an approximation of Birn-

baum's measure from:

$$I^{\mathrm{B}}(i) = \frac{I^{\mathrm{CR}}(i) \cdot Q_0}{q_i} \approx \frac{I^{\mathrm{VF}}(i) \cdot Q_0}{q_i}$$

B.6.4 System failure frequency obtained by $I^{\mathrm{B}}(i)$

An alternative way to calculate system failure frequency, F_0, is to start with Birnbaum's measure. First we recall that $I^{\mathrm{B}}(i)$ is the probability that the system is in such a state that component i is critical. That a component is critical means that the system is in such a state that the system is functioning if component i is functioning, and in a fault state if component i is failed. Then it follows that:

$$F_0 = \sum_i I^{\mathrm{B}}(i)(1 - q_i)\lambda_i \tag{B.18}$$

where $p_i = 1 - q_i$ is the probability that component i is functioning, and λ_i is the failure rate of component i. Thus, the contribution of component i to F_0 is given as the product of:

- The probability that component i is critical, i.e., the state of other components
- The probability that component i is functioning
- The failure rate of component i

Appendix C

Event Tree Analysis (ETA)

C.1 Introduction

An event tree is a logical diagram which displays possible event sequences following a specified critical event in a system. An event tree analysis (ETA) is a method for systematic analysis of a system after a critical event has occurred. The result of an ETA is a list of possible event sequences that follows the initiating event. The critical, initiating event may be a technical failure or some human error. In the development of the event sequences, the effects of possible barriers and safety functions, which are designed to prevent the occurrence of the critical event or reduce the consequences of this event, are taken into account. The analysis is both qualitative and quantitative. The qualitative content is primarily a visualisation of different scenarios (the event tree) with corresponding end consequences, while the quantitative analysis gives frequencies for the different end consequences. Figure C.1 shows an ETA example. The initial event could be for example SPAD = Signal passed at danger (obtained from for example an FTA), and then the various barriers are shown as B_1, B_2 etc. Each barrier has a Y=Yes output and a N=No output.

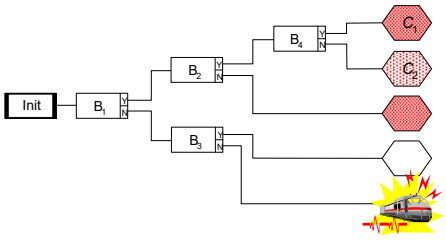


Figure C.1: ETA example

C.2 Procedure

The event tree analysis is usually carried out in six steps:

1. Identification of a relevant initiating event (which may give rise to unwanted consequences).
2. Identification of the barriers and safety functions which are designed to prevent the occurrence of the initiating event, or to reduce the consequences of this event.
3. Construction of the event tree.
4. Description of the resulting event sequences.
5. Calculation of probabilities/frequencies for the identified consequences.
6. Compilation and presentation of the results from the analysis.

C.3 Identification of a relevant initiating event

As for the fault tree it is important to define an unambiguous initiating event, use the "what", "where" and "when" keywords to structure the definition of the initiating event. How early in the course of event the initiating event should be placed depends on the scope and available resources for the analysis. As a starting rule we often define the initiating event as the first significant deviation from normal operations.

C.4 Identification of the barriers and safety functions

Usually, a number of measures are taken to prevent accidents or limit their consequences These measures are referred to by different names, e.g. "barriers", "security functions" or "protection layer" (defence in depth). The measures are modelled in the event tree. Other factors related to the physical course are also modelled, e.g.,

- Whether a leak ignites or not
- Whether the fire is large or small
- Whether it's day or night
- and so forth

C.5 Construction of the event tree

The event tree is constructed by thinking logical sequences by answering Yes/No questions. The questions should be formulated systematically. Either one uses consistent questions where the “Yes” answer is "success”, or Then the "Yes” answer is systematic failure or error in barrier/safety function The branches corresponding to "Yes” must either systematically go “upwards”, or systematically go “down” in the event tree.

If we adopt the convention that the “No” branch (“barrier fails to hold”) is the down-hand branch from the barrier symbol. The most severe consequences will then normally be located to bottom right corner of the consequence spectrum. Note that in some presentations “Yes” is used to describe that the barrier fails. This will then give a different interpretation of the most critical events.

If we consider a SPAD event, the first barrier, B_1, could be {Automatic train protection (ATP) OK}. When constructing the event tree the output from a barrier symbol may lead to another barrier symbol. The development is continued to the resulting consequences, illustrated by consequence symbols, C_1, C_2 etc in Figure C.1. We should aim at identifying the barriers in the sequence they are expected to be activated. In this way, there will be an implicit time line from left to right. However, in some situations this is demanding because it is not always easy to say which barriers are activated first.

C.6 Description of the resulting event sequences

The qualitative analysis of the event tree is typically to list the events leading up to the most severe end consequences, and discuss barriers and other circumstances that influence the course of events.

C.7 Calculation of probabilities/frequencies for the identified end consequences

In order to carry out the quantitative analysis we need the frequency of the initiating event, and the barrier probabilities. During construction of the event tree, we enter the probability that the various barriers fails, i.e., the “No” results. For each barrier, i, we need:

- q_i = probability that barrier i fails (“No”), and similarly
- $p_i = 1 - q_i$ probability that barrier i functions as intended (“Yes”)

In addition to the barrier probabilities, we enter the frequency of the initiating event:

- f = frequency of initiating event

When establishing the barrier probabilities and the initiating frequency it might be required to perform separate analyses, e.g., FTA. Also for the barrier probabilities we usually need separate analyses like FTA for the ATP system, failure statistics and "load/strength" methods.

To calculate the frequencies of the various consequences we may multiply the frequency of the initiating event by the barrier probabilities for each barrier along the path leading to the actual consequence . Now, consider consequence C_j, and assume that S = is the set of barriers in the path leading to consequence C_j, and that represents "success" of the barrier (Yes-terminal), and further F = is the set of those barriers on the path leading to consequence C_j, and that represent "the barrier fails" (No-terminal) we have that the frequency of consequence C_j is given by:

$$F_j = f \prod_{i \in S} p_i \prod_{i \in F} q_i$$

This formula is only valid if the barriers are "independent". This is not always the case, and to overcome the problem of "stochastic" dependent barriers, we should in principle specify the barrier probabilities as conditional probabilities given the course of events up to the current barrier. This is not always easy.

C.8 Combining FTA and ETA

Often, the event tree analysis is supplemented by doing a detailed fault tree analysis behind some of the "boxes" in the event tree:

- Behind the initiating event in an event tree there is often a fault tree analysis shedding light on the causes behind the initiating event
- Barriers in the event tree can often be modelled with a fault tree

As emphasized above formula (C.1) for the frequency of each end consequence assumes that the barriers are stochastically independent, and that the initiating event is independent of all barriers. If the initiating event and some of the barriers are modelled by fault trees, these fault trees might share some of the same basic event. In the modelling it is then possible to establish minimal cut sets for all end consequences. In this way we can explicitly treat dependencies in the ETA and FTA modelling. Appendix D a method for obtaining the cut sets is given.

C.9 Event Tree Analysis vs. Failure Tree Analysis

Fault- and event tree analyses are both used in risk analyses, but often with different purposes:

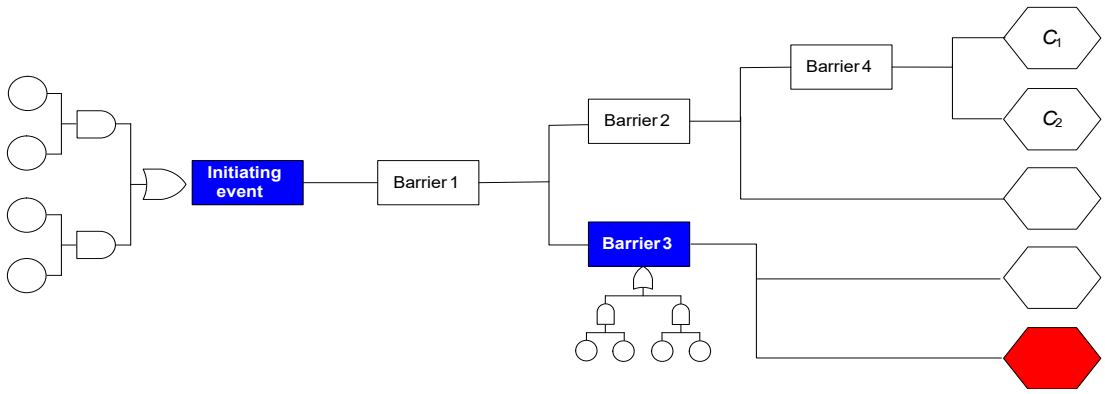


Figure C.2: Combining fault- and event trees

- A fault tree analysis is primarily used to analyse the causes of undesirable events, or the ca-uses of a failure of a barrier
  - For the fault tree analysis, there is only one "exit hand", i.e. the TOP event
  - Fault tree analysis is therefore out of the question if there are several outcomes
- We use the event tree analysis when we analyze the sequence of events after an unwanted event (i.e. the initiating event in the event tree)
  - An event tree analysis will have multiple outputs. Each branch in the event tree represents a possible outcome given that the undesirable event has occurred
  - Typically, we will draw an event tree for a system where several barriers are activated one by one after a critical situation
  - The severity of the sequence of events will typically increase the fewer barriers that work

Appendix D

Finding minimal cut sets in combined event- and fault tree systems

D.1 Introduction

This memo briefly describes how minimal cut sets may be obtained for combined event- and fault tree systems. It is assumed that the reader is familiar with the definition of cut sets in general, and how to obtain these.

D.2 Definitions

A *cut set* in a fault tree is a set of Basic events whose (simultaneous) occurrence ensures that the TOP event occurs. A cut set is said to be *minimal* if the set cannot be reduced without loosing its status as a cut set.

A *path set* in a fault tree is a set of Basic events whose non-occurrence (simultaneously) ensures that the TOP event does not occur. A path set is said to be *minimal* if the set cannot be reduced without loosing its status as a path set.

For small and simple fault trees, it is feasible to identify the minimal cut- and path sets by inspection without any formal procedure/algorithm. For large or complex fault trees we need an efficient algorithm. The MOCUS algorithm (Method for obtaining cut sets) is described in standard FTA textbooks, and an efficient improvement of the algorithm is described by Vatn (1993).

D.2.1 Dual fault tree

Let FT be a fault tree with basic events BE_i. A dual fault tree to FT, say FT*, is obtained by changing all AND gates in FT to OR gates, and all OR gates in FT to AND gates, and finally the basic

events of FT* are the complements of the corresponding basic events in FT.

D.2.2 Theorem

The minimal path sets of FT is given by the minimal cut sets of FT* with the complement basic events BE_i^* are replaced with BE_i.

D.3 Approach

This result will be useful if we have implemented an algorithm to find minimal cut sets, and if we need the minimal path sets. When we combine event- and fault trees this will be the case.

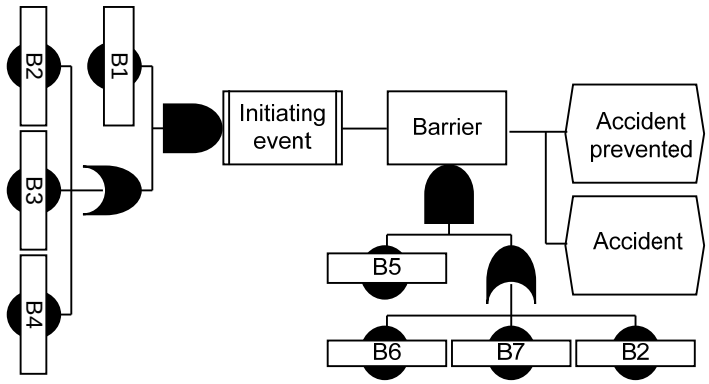


Figure D.1: Example system combining event and fault tree

Figure D.1 shows a system of combined fault and event trees. There is only one barrier in the event tree after the initiating event. To the left of the initiating event we have drawn a fault tree representing the combination of events that lead to the initiating event. Further below the barrier we have drawn a fault tree with the TOP event corresponding to the failure of the barrier. Two end consequences are drawn, one for "Accident prevented" corresponding to success of the barrier, and one for "Accident" corresponding to the failure of the barrier, i.e., the occurrence of the TOP event. Minimal cut sets for the left most fault tree are given by {B1,B2},{B1,B3}, and {B1,B4}. Minimal cut sets for the fault tree below the barrier are given by {B5,B6},{B5,B7}, and {B5,B2}.

Figure D.2 shows the dual fault tree of the rightmost fault tree in Figure D.1. The minimal cut sets of this dual fault tree is {B5*} and {B6*,B7*,B2*}. Thus the minimal path sets of the original fault tree is given by {B5} and {B6,B7,B2}. We observe that the occurrence of at least one cut set of the dual fault tree will ensure that the outcome of the barrier is a success. For example the occurrence of {B5*} corresponds to the non- occurrence of basic event B5.

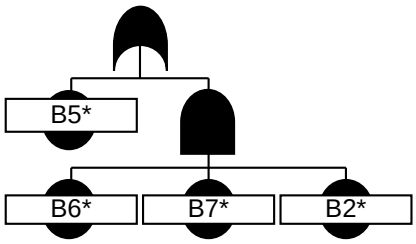


Figure D.2: Dual fault tree

In order to find the cut sets for each end consequence in the combined fault- and event tree system we need two types of cut sets operators, the & operator, and reduction operators. Let CSs1 and CSs2 be two cut sets, where CSs1 contains the minimal cut sets CS11, CS12, ..,CS1m, and CSs2 contains the minimal cut sets CS21,CS22,..,CS2n. We now define the &-operator for two set of cut sets:

&-operator

The &-operator for two set of cut sets is defined such that CSs3 = CSs1 & CSs2 is a new set of cut sets where CSs3 is the set of all combination of minimal cut sets from CSs1 and CSs2. A combination of two cut sets CSa and CSb in this context is the set of all events in CSa and all events in CSb (the union of events in each of them). Note that CSs3 might contain non-minimal cut sets. Also note that each cut set contained in CSs3 may contain repeated evens, and also events that may not occur simultaneously. After applying the &-operator we need post-processing of the result. Three types of reductions are necessary, (i) eliminate repeating events, (ii) remove cut sets with two or more events that may not occur simultaneously, and (iii) eliminate non minimal cut sets. The idea for finding cut sets for each end consequence we collect relevant cut sets along the paths from the initiating event to the various end consequences. Note that if a fault tree is not developed for the initiating event or a barrier the cut sets only contain one cut set, and this one again is only one event, either (i) the initiating event, or (ii) the success, or (iii) failure of the barrier

D.4 Procedure

The procedure for finding the minimal cut sets for one end consequence is now as follows where CSs is the running set of cut sets:

1. Start with the initiating event. If a fault tree is developed for the initiating event, let CSs be the corresponding set of minimal cut sets, else the set of cut sets is the initiating event it self.

2. Proceed with the next barrier along the path until the required end consequence is reached.
3. If the barrier along the path we are following corresponds to a failure of this barrier, let CSsB be the set of cut sets for the fault tree of the barrier, or let CSsB be the barrier failure it self if no fault tree is developed. If we are following the success of this barrier let CSsB be the minimal cut set of the dual fault tree, or if no fault tree exists for this barrier, let CSsB be the complement of the barrier failure, i.e., the barrier success.
4. Apply the & operator, i.e., CSs ← CSsB & CSs
5. Remove the second of repeated events in each cut set of CSs
6. Remove cut sets of CSs containing a basic event, say B_i, and it's complement B_i^*.
7. Remove non-minimal cut sets in CSs
8. GoTo Step 2
9. When the end consequence is reached, CSs now is the set of minimal cut sets for this event

We now demonstrate the procedure for the example in Figure D.1, and we start with the "accident" end consequence.

In Step 1 we find CSs = { {B1,B2},{B1,B3},{B1,B4} }. In Step 3 we find CSsB = { {B5,B6},{B5,B7}, {B5,B2} }. By applying the & operator we get:

CSs = { {B1,B2,B5,B6}, {B1,B2,B5,B7}, {B1,B2,B5,B2},{B1,B3,B5,B6}, {B1,B3,B5,B7}, {B1,B3,B5,B2}, {B1,B4,B5,B6},{B1,B4, B5,B7},{B1,B4, B5,B2} }.

In Step 5 we remove one occurrence of B2 where it occurs twice, and get:

CSs = { {B1,B2,B5,B6}, {B1,B2,B5,B7}, {B1,B2,B5}, {B1,B3,B5,B6}, {B1,B3,B5,B7}, {B1,B3,B5,B2}, {B1,B4,B5,B6}, {B1,B4,B5,B7}, {B1,B4,B5,B2} }

Neither {B1,B3,B5,B2} nor {B1,B4,B5,B2} are minimal cut sets since {B1,B2,B5} is a minimal cut set, hence these two cut sets are removed in Step 7, and we remain with:

CSs = { {B1,B2,B5,B6}, {B1,B2,B5,B7}, {B1,B2,B5}, {B1,B3,B5,B6}, {B1,B3,B5,B7}, {B1,B4,B5,B6}, {B1,B4,B5,B7}}

These are the minimal cut sets since there are no occurrences of an event and its complement in any of the cut sets, and there are no non-minimal cut sets left.

Proceeding to the “accident prevented” end consequence, we have:

In Step 1 we find CSs = { {B1,B2},{B1,B3},{B1,B4} }. In Step 3 we find for the dual fault tree CSsB = { {B5*},{B6*,B7*,B2*} }. By applying the & operator we get CSs = { {B1,B2,B5*}, {B1,B2,B6*,B7*,B2*}, {B1,B3,B5*}, {B1,B3,B6*,B7*,B2*},{B1,B4,B5*}, {B1,B4,B6*,B7*,B2*} }.

In Step 6 we see that {B1,B2,B6*,B7*,B2*} never will occur since B2 and B2* cannot occur at the same time, hence this cut set is removed, and we remain with:

CSs = { {B1,B2,B5*}, {B1,B3,B5*}, {B1,B3,B6*,B7*,B2*},{B1,B4,B5*}, {B1,B4,B6*,B7*,B2*} }

Note that the minimal cut sets only cover the situation where the initiating event occurs. Finding the dual fault tree to the left most fault tree could also find minimal cut sets for “nothing”, but this is usually not of interest, and hence omitted.

Appendix E

Including conditional probability tables in a fault tree

E.1 Introduction

Chapter 4 describes how a fault tree can be converted to a BBN. Such an approach enables to convert the hybrid Risk_OMT model into a full BBN. For large systems this might cause memory problem in order to represent the system. Further the BBN approach is not able to present the minimal cut sets. Another challenge with BBN is that a fault tree represented by a BBN can only handle the probability of the TOP-event, i.e., Q_0, but it is not straight forward to handle the frequency of the TOP event, i.e., F_0.

On the other hand, an attractive feature of a BBN is that we do not need to stick to strict logical gates. In this Appendix we propose an approach where we can implement BBN-features in the FTA.

E.2 Motivating example

Consider a ship with two propellers powered by one engine. Under normal conditions it is considered that the ship can operate safe with one propeller. Figure E.1 shows the fault tree.

An AND gate is used to represent that both propellers need to fail in order to cause the propellers system to fail. In some situations we might need both propellers, and even in some sever situations we can not do with both propellers failing. In a BBN we would have represented this as a conditional probability table (CPT) as shown in Table E.1. Her T (True) means a fault state, and F (false) means a functioning state.

The CPT can also be viewed as a representation of four disjoint scenarios. For example the first row (T-T) states that if both P1 and P2 are failed, the propellers system will fail with 80%

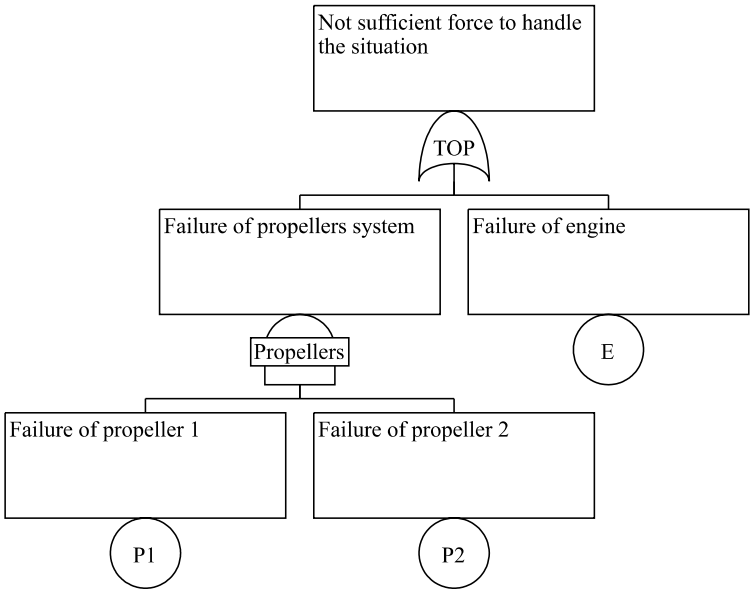


Figure E.1: Fault tree for the ship

Table E.1: Conditional probability table for the propeller gate

Probability	P1	P2
0.80	T	T
0.10	T	F
0.08	F	T
0.02	F	F

probability etc. Since the CPT represent disjoint situations, we may represent they in a fault tree by introducing an OR-gate that combines the relevant combinations. To simplify, we ignore the F-F situation which has a low value in the CPT. The input to the OR-gate is AND-gates for each scenario. One input then represent the probability that this scenario is relevant (to the left), and the other input is another AND-gate representing the combinations. The syntax is to put a star behind the basic event name to represent the dual event. For example for scenario 2 (T-F) the basic events are P1 and P1*.

Figure E.2 shows the corresponding fault tree. The minimal cut sets are:

```
{E}
{P_TT,P2,P1}
{P_TF,P2*,P1}
{P_FT,P1*,P2}
```

The basic event P_TT is then representing the situation that both propellers need to fail in order to let the original AND-gate be True. The basic event probability should then be set to 80%.

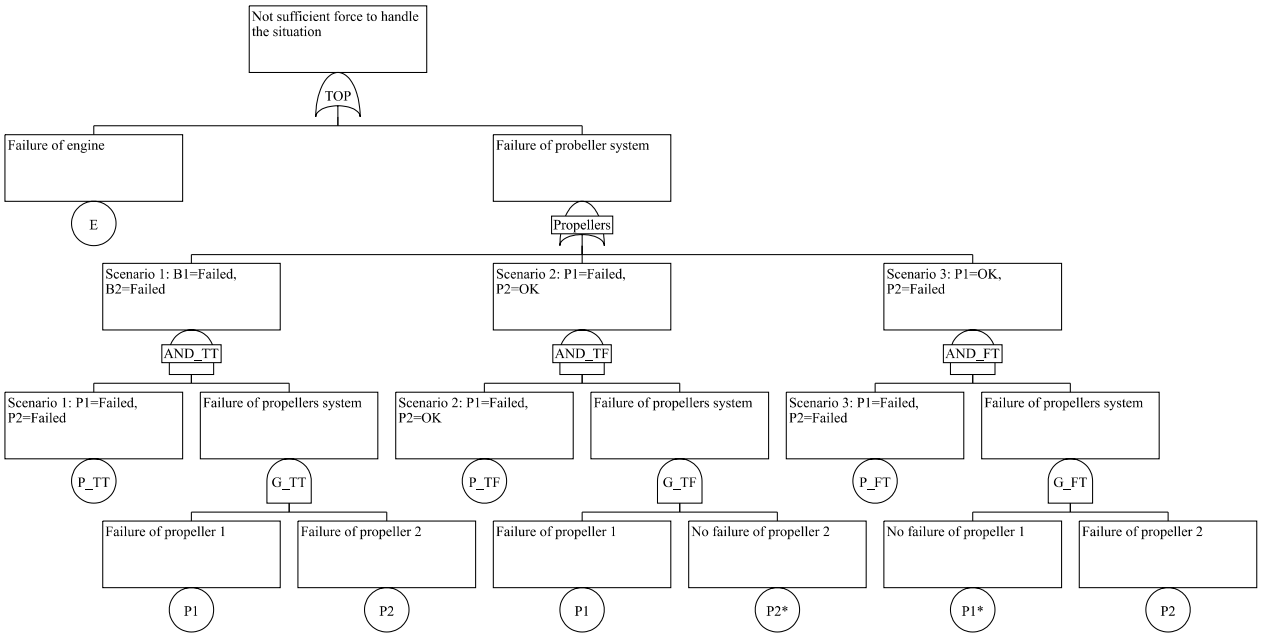


Figure E.2: Fault tree for the ship with AND gate replaced with a sub-tree representing the CPT

The basic event probabilities for the dual basic events is given by one minus the probability of the corresponding basic event.

E.3 Method

To generalize the method we start with the situation where the "pivot"-gate only has basic events as input. Let this gate be dented the CPT-gate, i.e., a gate that shall be converted to a sub-tree representing the CPT. The CPT must then be specified in a table similar to Table E.1. The probabilities should add up to one. Some combinations might not be relevant. For example with the ship example we might consider that there is never a situation where the functioning of both propeller will cause the propeller system to fail.

Step 1 - Replace CPT-gate

The CPT-gate is replaced with an OR-gate. The number of inputs to the OR-gate is the number of rows in the corresponding CPT-table.

Step 2 - Insert sub-trees for each input to the OR-gate

An AND-gate is used to represent that the "scenario" is occurring AND that the corresponding combination of the input to the CPT-gate is occurring. The occurrence of the scenario is repre-

sented by a new basic event. The combination of the input to the CPT-gate is then represented by an AND-gate with basic events or dual basic events are used to match the corresponding T and F values. Figure E.2 shows an example.

Step 3 - Find the minimal cut sets

A standard method like MOCUS can be used to find the minimal cut sets.

Step 4 - TOP-event quantification

When we shall calculate Q_0 we use standard methods like the upper-bound approximation. Probabilities for the basic event probabilities representing each scenario is given by the CPT. Probabilities for the input events to the CPT-gate are found by standard formulas for basic event probabilities, typically $q = \lambda$MDT. Probability for the dual events are given by one minus the non-dual probability, typically $q = 1 - \lambda$MDT.

When calculating the frequency of the TOP-event, i.e., F_0 we add the contribution from each cut set. Equation (B.9) is a starting point for such a calculation. Note that when we sum over the ω_i-values there is no "rate" of the *dual* events.

E.4 Input to the CPT-gate represent sub-trees

The method described in section E.3 assumes that the inputs to the CPT-gate are all basic events. In some situations some of the inputs to the CPT-gate are sub-trees. To cope with this the following approach may be use:

1. Follow the approach in section E.3
2. if one or more inputs to the CPT-gate are sub-trees, replace each sub-trees with a temporary basic event and dual event
3. Find the minimal cut-sets with these temporary basic events. Let these cut sets be denoted MCSs
4. For each sub-tree, find the minimal cut sets for that sub-tree, and the corresponding dual sub-tree
5. Replace the temporary basic events in MCSs with the corresponding cut sets, and repeat for the dual events
6. Remove duplicates in each cut set of the new MCSs

7. Remove non-feasible cut sets, i.e., cut sets where both a basic event and its dual event occur in the same cut set

8. Remove non-minimal cut sets

Note that replacing a temporary basic event with the corresponding cut sets for the sub-tree means that the relevant cut sets will be expanded "in both directions" where we get a new cut set for each of the cut sets of the sub-tree. For example, assume the following MCSs for the "main" fault tree:

```
{B_TT,B1,Sub}
{B_TF,B1,Sub*}
{B_FT,B1*,Sub}
```

Further assume that the Sub sub-tree has one AND gate with basic events B2 and B3 leading to the minimal cut set:

```
{B2,B3}
```

and dual fault tree Sub* has the minimal cut set:

```
{B2*}
{B3*}
```

Inserting this gives:

```
{B_TT,B1,B2,B3}
{B_TF,B1,B2*}
{B_TF,B1,B3*}
{B_FT,B1*,B2,B3}
```

Appendix F

Reliability of Safety Systems

F.1 Introduction

The international standard Functional safety of electrical/electronic/programmable electronic safety-related systems (IEC 61508, 2010) is a generic, performance-based standard for safety-related systems. In this chapter we present some fundamental elements addressed in IEC 61508.

F.1.1 Safety Instrumented Systems and Safety Instrumented Functions

A safety instrumented system (SIS) is an independent protection layer that is installed to mitigate the risk associated with the operation of a specified hazardous system, which is referred to as the equipment under control (EUC). An example of an EUC is a process vessel.

A SIS is composed of *sensors* often referred to as *input elements*, *logic solvers* and *actuating items* often referred to as *final elements*.

A safety instrumented function (SIF) is a function that is implemented by a SIS and that is intended to achieve or maintain a safe state for the EUC with respect to a specific process demand such as high pressure in the vessel.

A SIS has two main system functions:

1. When a predefined process demand occurs in the EUC; the deviation shall be detected by the SIS sensors, and the required actuating items shall be activated and fulfil their intended functions.

2. The SIS shall not be activated spuriously, that is, without the presence of a predefined process demand in the EUC.

A *demand* is defined as: An event or a condition that requires a SIF to be activated (i) to prevent an undesired event from occurring or (ii) to mitigate the consequences of an undesired event. In the process industry, a demand is also called a *process upset* or a *process deviation*.

F.1.2 Testing of Safety Instrumented Functions

A SIS is often a passive system that is activated only when a demand occurs. Failures may therefore occur and remain hidden until the system is demanded or tested. We often referred to two main categories of testing.

- *Proof Testing.* To verify that a SIS is able to perform its SIFs, the system is usually proof tested at regular intervals of length τ. The time interval between two consecutive proof tests is often called the proof test interval. Proof testing is also called functional testing. Dangerous failures detected by proof testing are called dangerous undetected (DU) failures.

- *Diagnostic testing.* A diagnostic test is an automatic partial test that uses built-in self-test features to detect failures. Dangerous failures detected by a diagnostic test are called dangerous detected (DD) failures. The identified faults are announced as alarms, locally at the equipment and in the control room.

F.1.3 Safety Integrity Levels (SILs)

IEC 61508 uses safety integrity as a performance measure for a SIF. Safety integrity is the probability of a SIS satisfactorily performing the specified SIFs under all the stated conditions within a stated period of time. IEC 61508 does not specify detailed probability values, but divides the requirements into four safety integrity levels, SIL 1, SIL 2, SIL 3, and SIL 4, with SIL 4 being the most reliable and SIL 1 being the least reliable.

F.1.4 Reliability Metrics

Probability of Failure on Demand

The probability of (dangerous) failure on demand, PFD(t) is the probability that the SIS has a dangerous fault and that it is not able to perform its SIFs at time t. The notion probability of failure on demand may indicate that we are dealing with a conditional probability, given that a demand has occurred. This is not correct and PFD(t) may be expressed as

$$\Pr(\text{The SIS is not able to perform its SIF at time } t)$$

irrespective of whether a demand occurs or not. If a demand should occur at time t, PFD(t) is the probability that the SIS fails to perform its SIF. In many cases, it is not necessary to determine the PFD as a function of time and we can suffice with an average value. If the SIF is proof tested after regular intervals of length τ and the system is considered to be as-good-as-new after each

proof test, the long-term average probability of failure on demand can be expressed as

$$\text{PFD} = \frac{1}{\tau}\int_0^\tau \text{PFD}(t)\,dt$$

Average Frequency of Dangerous Failures per Hour

For SIFs that are operated in high-demand or continuous mode, IEC 61508 requires that the reliability is specified by the average frequency of dangerous failures (PFH) where the frequency is given as number of dangerous failures per hour. The abbreviation PFH is retained from the previous version of IEC 61508 where the metric was called "average probability of (dangerous) failure per hour." The idea behind using the PFH as a reliability metric is that demands will occur so often that when a dangerous failure of the SIF occurs, it is most likely that a demand will occur and a hazardous event will be manifested before we can bring the EUC to a safe state.

Spurious Trip Rate and related concepts

The spurious trip rate, (STR) is the rate of spurious trips of a specified SIF per hour.

There are three main types of spurious activation: (i) spurious activation of individual components, (ii) spurious activation of a SIF, and (iii) spurious shutdown of the process. To use the same concept to describe all the three types may lead to misunderstanding and confusion. To distinguish the different types of spurious activation, the following terms and definitions are often used (deviates from the presentation in the textbook):

- *Spurious operation.* A spurious operation (SO) is an activation of the safety function of a channel without the presence of a specified process demand. A spurious operation of a channel is said to be an SO-failure and the SO-failure rate is denoted λ_{SO}.
- *Spurious trip.* A spurious trip (ST) is an activation of a SIF without the presence of a specified process demand.
- *Spurious shutdown.* A spurious shutdown is a partial or full process shutdown without the presence of a specified process demand.

Reliability Metrics and SIL

To fulfil the requirements of a safety integrity level, a SIF in low-demand mode must have a PFD in the corresponding interval specified in Table F.1. Similarly, a SIF in high-demand or continuous mode must have a PFH in the corresponding interval specified in Table F.1.

Table F.1: SIL requirements vs PFD/PFH

SIL	**Low demand mode of operation** (Average probability of failure to perform its design function on demand)	**High demand mode of operation** (Average probability of failure per hour to perform its design function)
4	$10^{-5} \leqslant$ PFD $<10^{-4}$	$10^{-9} \leqslant$ PFH $<10^{-8}$
3	$10^{-4} \leqslant$ PFD $<10^{-3}$	$10^{-8} \leqslant$ PFH $<10^{-7}$
2	$10^{-3} \leqslant$ PFD $<10^{-2}$	$10^{-7} \leqslant$ PFH $<10^{-6}$
1	$10^{-2} \leqslant$ PFD $<10^{-1}$	$10^{-6} \leqslant$ PFH $<10^{-5}$

Classification of Failures Based on Consequence and Detectability

Hardware failures can be classified as:

- *Dangerous* (D) failure. A dangerous failure is a failure that brings the item into a state where it is not able to perform its safety function(s). When the item is in such a state, it is said to have a dangerous (D) fault.
- *Safe* (S) failure. A safe failure is a failure that does not leave the item in a state where it is not able to perform its safety function(s). When the item is in such a state, it is said to have a safe (S) fault.

Dangerous and safe hardware failures/faults may also be categorized as detected or undetected.

- *Detected.* A fault that is detected by automatic diagnostic testing, internal in the item or connected to a logic solver.
- *Undetected.* A fault that is not detected (not diagnosed) by automatic diagnostic testing, internal in the item or connected to a logic solver.

Combining the two principles of categorization yields:

- *Dangerous undetected* (DU) faults. DU-faults are preventing activation on demand and are revealed only by proof testing or when a demand occurs. DU-faults are sometimes called dormant or hidden faults. The DU-faults are of vital importance when calculating the SIF reliability as they are a main contributor to SIF unavailability.
- *Dangerous detected* (DD) faults. DD-faults are detected short time after they occur, by automatic diagnostic testing. The average period of unavailability due to a DD-failure is called the mean time to restoration (MTTR), the mean time elapsing from the DD-failure occurs until the function is restored.

- *Safe undetected* (SU) failures. Non-dangerous failures that are not detected by automatic self-testing.
- *Safe detected* (SD) failures. Non-dangerous failures that are detected by automatic self-testing. In some configurations, early detection of failures may prevent an actual spurious trip of the system.

F.1.5 PFD calculations for systems

To obtain PFD for a system we may follow the following procedure:

1. Find PFD for the system as a function of t in an interval, i.e., $0 \leqslant t \leqslant \tau$, and denote the result PFD(t)
2. To obtain PFD(t) we often utilize the system survivor function, say $R(t)$
3. Find the average PFD(t) by integration: PFD $= \frac{1}{\tau}\int_0^\tau \text{PFD}\,(t)\,dt = 1 - \frac{1}{\tau}\int_0^\tau R\,(t)\,dt$

The classical example is one component proof tested at point of times τ,2τ,3τ,..., and time to failure is exponentially distributed, i.e., $R(t) = e^{-\lambda t}$.

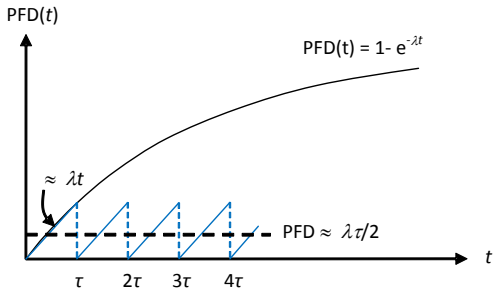
$$\text{PFD} = 1 - \frac{1}{\tau}\int_0^\tau R(t)dt = 1 - \frac{1}{\tau}\int_0^\tau e^{-\lambda t}dt = 1 + \frac{1}{\lambda\tau}\int_0^\tau -\lambda e^{-\lambda t}dt = 1 + \frac{1}{\lambda\tau}e^{-\lambda t}\Big|_0^\tau = 1 + \frac{1}{\lambda\tau}\left(e^{-\lambda\tau} - 1\right)$$

If $\lambda\tau$ is small, i.e., (<0.01) we utilize that $e^{-x} \approx 1 - x + x^2/2$, and inserting in the expression for PFD(t) yields:

$$\text{PFD} = 1 + \frac{1}{\lambda\tau}\left(e^{-\lambda\tau} - 1\right) \approx 1 + \frac{1}{\lambda\tau}\left(1 - \lambda\tau + (\lambda\tau)^2/2 - 1\right) = \lambda\tau/2$$

Note that λ is the rate of DU-failures. In some presentations the notation λ_{DU} is used, but for simplicity we only use λ. In general we allso need to add contribution of DD failures, but this is not further discussed in this presentation.

Another way to obtain the same result is to use that $e^{-x} \approx 1 - x$ for small x-values directly, hence we have that PFD(t) $= 1 - e^{-\lambda t} \approx \lambda t$ in an each proof test interval:



which yields PFD $\approx \lambda\tau/2$.

Such an argument we may also use for two identical components in parallel that are proof tested at the same time. The time dependent PFD of the two components is found by

$$\text{PFD}(t) = \text{PFD}_1(t) \cdot \text{PFD}_2(t) \approx (\lambda t)^2$$

yielding:

$$\text{PFD} = 1/\tau \int_0^\tau \text{PFD}_1(t) \cdot \text{PFD}_2(t)\,dt \approx 1/\tau \int_0^\tau (\lambda t)^2 dt = \frac{(\lambda\tau)^2}{3}$$

The PFDs of some $koon$ systems of identical and independent components with constant failure rate λ and test interval τ are found to be:

$k\backslash n$	1	2	3	4
1	$\frac{\lambda\tau}{2}$	$\frac{(\lambda\tau)^2}{3}$	$\frac{(\lambda\tau)^3}{4}$	$\frac{(\lambda\tau)^4}{5}$
2	–	$\lambda\tau$	$(\lambda\tau)^2$	$(\lambda\tau)^3$
3	–	–	$\frac{3\lambda\tau}{2}$	$2(\lambda\tau)^2$
4	–	–	–	$2\lambda\tau$

The general formula for PFD is

$$\text{PFD} = \binom{n}{n-k+1} \frac{(\lambda\tau)^{n-k+1}}{n-k+2}$$

The argument for this formula is as follows. We have a $k\,oo\,n : G$ system corresponding to an $n-k+1\,oo\,n : F$ system. This means that if $n-k+1$ components are in a fault state, the system will be in a fault state. The minimal cut sets will all contain $n-k+1$ components, and there are

$\binom{n}{n-k+1}$ such minimal cut sets. At time $t, 0 \le t < \tau$ the probability that one such cut set is in a fault state is $\text{PFD}_j(t) \approx (\lambda t)^{n-k+1}$ with an average $\text{PFD}_j = \frac{(\lambda\tau)^{n-k+1}}{n-k+2}$. Multiplying with the number of minimal cut sets gives the above formula.

F.1.6 Common cause failures

The equation above assumes that components in a SIS fail independent of each other. In practice components may fail due to common causes. Common cause failure may be due to maintenance introduced failures, design failures, excessive stress etc. To model common cause failures the total failure rate of one component (i.e., rate of DU failures) is split into an independent part and a dependent part:

$$\lambda = \lambda_{(i)} + \lambda_{(c)} = (1-\beta)\lambda + \beta\lambda$$

where $\beta = \lambda_{(c)}/\lambda$ is the common cause factor. This (beta factor) model now yields:

- For the dependent part, use PFD = $\frac{\beta\lambda\tau}{2}$
- For the independent part, use the independent failure rate $(1-\beta)\lambda$ in the PFD formulas of the $k$oo$n$ system of identical and independent components
- Add the contributions:

$$\text{PFD} = \frac{\beta\lambda\tau}{2} + \binom{n}{n-k+1}\frac{\left[(1-\beta)\lambda\tau\right]^{n-k+1}}{n-k+2}$$

Staggered Testing

Now, consider the case with the two components in a 1oo2 voting having the same λ and τ, but where the testing is not carried out simultaneously. The situation is illustrated in Figure F.1.

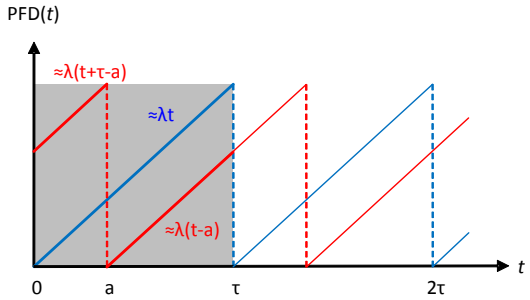


Figure F.1: Staggered testing

Assuming that component 2 is tested at time (a) inside the test interval of component 1, it can be shown that:

$$\text{PFD}(a) \approx \frac{(\lambda\tau)^2}{3}\left(1 - \frac{3a}{2\tau} + \frac{3a^2}{2\tau^2}\right)$$

PFD(a) attains its maximum value

$$\text{PFD}_{max} \approx \frac{(\lambda\tau)^2}{3} = \frac{4}{3} \cdot \frac{\lambda\tau}{2} \cdot \frac{\lambda\tau}{2}$$

when $a = 0$ or $a = \tau$, i.e., when the components are tested simultaneously.

PFD(a) attains its minimum value when $a = \tau/2$, i.e., when component 2 is tested in the middle of the test interval of component 1:

$$\text{PFD}_{min} \approx \frac{5}{8}\frac{(\lambda\tau)^2}{3} = \frac{5}{6} \cdot \frac{\lambda\tau}{2} \cdot \frac{\lambda\tau}{2}$$

Note that this minimum PFD is actually smaller than the PFD obtained when simply multiplying the average PFD values of the individual components. Compared to the case of simultaneous testing, we obtain a PFD reduction of 38% in the case of "optimal" testing. Hence, there is a great potential for improvement in the total PFD if components are tested at different times. This is exploited in staggered testing. Also note that the minimum value is obtained when $a = \tau/2$ for a 1oo2 system, for general configuration it is more complicated to set up the optimal staggered testing regime.

F.1.7 More about the test regime

There are three different test regimes that are considered

- Simultaneous testing, i.e., $a = 0$ in Figure F.1
- Optimal staggered testing, i.e., $a = \tau/2$ in Figure F.1
- Independent testing

If the components are tested independently we can calculate PFD for each component by the formula $\text{PFD}_i = \lambda_i\tau_i/2$ and proceed with the structure function. Due to the independent test regime, it is reasonable to argue that the components are independent, and we proceed with the standard approach which here means to replace x_i in the structure function with $p_i = 1 - \text{PFD}_i$. If common cause failures are relevant, we may add an artificial block to represent the common cause "part" of the components.

It is important to understand the difference between independent *testing* and independent *components*. Independent *testing* means that if we know that one of the component is in a fault state this will have no information regarding if the other component is in a fault state. This is not true for e.g., simultaneous testing. For simultaneous testing we have: If one component is known to be in a fault state it is more likely that we are at the end of the test interval compared to if it was functioning. Hence, it is also more likely that the other component is in a fault state because the likelihood of being at the end of the test interval is higher. The components performance are dependent not because of any physical reasons, only due to the testing regime.

F.1.8 The PDS method

The PDS method is developed by SINTEF Safety. The method has two main features:

1. It proposes a more realistic way to model common cause failures (CCF). In the β-factor model a CFF will always cause all components to fail. This is often not realistic. The PDS method therefore proposes a correction factor to adjust the β-factor to account for the situation where not all components fail due to the CCF situation.

2. In IEC 61508 only random hardware failures are quantified. The PDS method also quantifies systematic failures by the so-called test independent failure term, p_{TIF}. Systematic failures are not treated in this presentation.

The idea behind adjusting the β-factor is that if we have a CFF causing two components to fail, it is not certain that the remaining components will fail. Some assumptions are then made regarding the probability that a third component fails given that two components have failed due to a CFF and so on. The correction factor is dependent on k and n, and is generally denoted C_{koon}, and is presented in Table F.2 for some combinations:

Table F.2: C_{koon} correction factors

$k \backslash n$	$n=2$	$n=3$	$n=4$	$n=5$	$n=6$
$k=1$	$C_{\text{1oo2}}=1.0$	$C_{\text{1oo3}}=0.5$	$C_{\text{1oo4}}=0.3$	$C_{\text{1oo5}}=0.20$	$C_{\text{1oo6}}=0.15$
$k=2$	-	$C_{\text{2oo3}}=2.0$	$C_{\text{2oo4}}=1.1$	$C_{\text{2oo5}}=0.8$	$C_{\text{2oo6}}=0.6$
$k=3$	-	-	$C_{\text{3oo4}}=2.8$	$C_{\text{3oo5}}=1.6$	$C_{\text{3oo6}}=1.2$
$k=4$	-	-	-	$C_{\text{4oo5}}=3.6$	$C_{\text{4oo6}}=1.9$
$k=5$	-	-	-	-	$C_{\text{5oo6}}=4.5$

A configuration specific β-factor is now calculated by multiplying the original β-factor with the correction factor C_{koon} found in Table F.2. Note that the baseline β-factor is assumed to be specified for a 1oo2 system.

F.1.9 PFH calculations

In the following we present the simple approximation formula proposed in the PDS method for the probability of failure per hour, PFH:

$$\text{PFH} = C_{\text{koon}}\beta\lambda + \frac{n![\lambda(1-\beta)\tau]^{n-k+1}}{(n-k+1)!(k-1)!\tau} \tag{F.1}$$

Note that the correction factor C_{koon} only applies for the PDS method. To obtain eq. (F.1) we treat CCF failures and independent failures individually. For the CCF failures the results is rather obvious. For independent failures, let $p(t,k,n)$ be the probability that the first $n-k$ components are in a fault state assuming they are numbered $1,2,\ldots,n$. If the the first $n-k$ components are in a fault state and the remaining k components are functioning, then they are *critical*. A system failure will occur if one of remaining k components fails. We have that $p(t,k,n) \approx \left[\lambda(1-\beta)t\right]^{n-k}$. To find the contribution to the PFH for this situation we calculate the average of $p(t,k,n)$ in a proof test period, and multiply with $f_k = k\lambda(1-\beta)$. f_k is the total frequency of the event that one of the remaining k components fails. The above argument is valid when we consider the numbered $n-k$ components. There are $\binom{n}{n-k} = \frac{n!}{(n-k)!k!}$ ways to chose $n-k$ components, and adding up we obtain eq. (F.1). Note that the probability that the k remaining components being in a *functioning* state is considered to be close to one, so we do not take this into account.

F.1.10 STR calculations

In the following we present the simple approximation formula proposed in the PDS method for the spurious trip rate, STR:

$$\text{STR} = C_{(\text{n-k+1})\text{oon}}\beta\lambda_{\text{SU}}$$

Note that the correction factor $C_{(\text{n-k+1})\text{oon}}$ only applies for the PDS method. We have here explicitly indicated that the failure rate to go into the formula is the rate of SU failures. In some presentations λ_{SO} is used to reflect the rate of spurious operations on component level.

F.1.11 Markov approach

This presentations deviates from the presentation in the textbook. We have seen that the Markov equations may be written on matrix form:

$$\mathbf{P}(t)\cdot\mathbf{A} = \dot{\mathbf{P}}(t)$$

which may be approximated by:

$$\dot{\mathbf{P}}(t) = \frac{\mathbf{P}(t+\Delta t) - \mathbf{P}(t)}{\Delta t} = \mathbf{P}(t) \cdot \mathbf{A}$$

yielding

$$\mathbf{P}(t+\Delta t) = \mathbf{P}(t)[\mathbf{A}\Delta t + \mathbf{I}]$$

where $\mathbf{I}$ is the identity matrix. This equation may now be used iteratively with a sufficient small time interval Δt and starting point $\mathbf{P}(0)$ to find the time dependent solution. Only simple matrix multiplication is required for this approach.

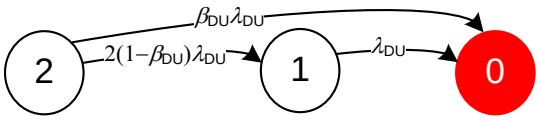
PFD

Assume that we know the state vector $\mathbf{P}(0)$ just after a proof test, and that we have established a Markov transition model for the SIS with respect to a given SIF. Then it is straight forward to find $\mathbf{P}(t)$ within a proof test interval by the approach presented above. Typically the probability of being in a state where all components are functioning (state r) is assumed to be one, and probabilities for the other states are equal to zero. Let F be the set of failed states with respect to the actual safety function of the SIS. We then have:

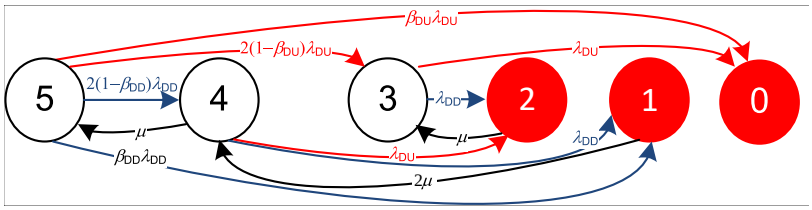
$$\text{PFD} = \frac{1}{\tau}\int_0^\tau \text{PFD}(t)\,dt = \frac{1}{\tau}\int_0^\tau \sum_{i\in F} P_i(t)\,dt$$

The integral is replaced by a sum in the numerical calculations since we are already solving the time dependent solution iteratively by time steps Δt.

The following figure shows the Markov diagram for a 1oo2 system considering DU-failures only:



Note that whereas the closed form formulas for PFD presented earlier only takes DU failures into account. With the Markov approach, DD failures may also be included. The Markov diagram for the 1oo2 systems now reads:



where the following system states are defined:

5: Both components OK

4: One OK, one DD-failure

3: One OK, one DU-failure

2: One DU-failure and one DD-failure

1: Two DD-failures

0: Two DU-failures

Red arrows are DU failures, blue arrows are DD failures, and black arrows are repairs.

PFH

The procedure is now similar to the approach for PFD, but we are seeking a rate, i.e., the rate of transition from a functioning state to a fault state for the SIS safety function is found by averaging:

$$\text{PFH} = \frac{1}{\tau}\int_0^\tau \text{PFH}(t)dt = \frac{1}{\tau}\int_0^\tau \sum_{i\notin F}\sum_{j\in F} a_{ij}P_i(t)dt$$

where a_{ij} is the transition rate from state i to state j measured in expected number of transitions *per hour*. Note that we can interchange the integration and summation operators, i.e., we may first calculate the average state probabilities, then calculate the appropriate transition rates.

STR

The procedure for the spurious trip rate is now similar to the approach for PFH, but we need to consider the spurious trip system failure mode. Therefore, we typically need to draw a new Markov diagram. Let F be the set of system failure states representing a spurious trip state. STR is found by averaging:

$$\text{STR} = \frac{1}{\tau}\int_0^\tau \text{STR}(t)dt = \frac{1}{\tau}\int_0^\tau \sum_{i\notin F}\sum_{j\in F} a_{ij}P_i(t)dt$$

Bibliography

Apostolakis, G. (2004). How useful is quantitative risk assessment? *Risk analysis : an official publication of the Society for Risk Analysis*, 24:515–20.

Cox, D. (1972). Regression models and life-tables. *Journal of the Royal Statistical Society, Series B.*, 34 (2):187–220.

Endsley, M. R. (1995). Toward a theory of situation awareness in dynamic systems. *Human Factors*, 137(1):32–64.

Endsley, M. R. (2015). Situation awareness misconceptions and misunderstandings. *Journal of Cognitive Engineering and Decision Making*, 9(1):4–32.

Gran, B., Bye, R., Nyheim, O., Okstad, E., Seljelid, J., Sklet, S., Vatn, J., and Vinnem, J. (2012). Evaluation of the risk omt model for maintenance work on major offshore process equipment. *Journal of Loss Prevention in the Process Industries*, 25(3):582–593.

Hollnagel, E., Woods, D., and Leveson, N. (2006). Resilience engineering : Concepts and precepts. *Resilience Engineering: Concepts and Precepts*.

Mohaghegh, Z., Kazemi, R., and Mosleh, A. (2009). Incorporating organizational factors into probabilistic risk assessment (pra) of complex socio-technical systems: A hybrid technique formalization. *Reliability Engineering & System Safety*, 94(5):1000–1018.

Reason, J. (1990). *Human Error*. Cambridge University Press.

Sklet, S., Vinnem, J. E., and Aven, T. (2006). Barrier and operational risk analysis of hydrocarbon releases (bora-release): Part ii: Results from a case study. *Journal of Hazardous Materials*, 137(2):692–708.

Vatn, J. (2012). Can we understand complex systems in terms of risk analysis? *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, 226(3):346–358.

Vatn, J., de Lauzon, J., Pérez Morán, G., and Auer, G. (2012). Overall methodology for risk assessments. Standard, Commision of the European communities.

Vatn, J. and Haugen, S. (2012). On the usefulness of risk analysis in the light of deepwater horizon and gullfaks c. In Besnard, D. and Albrechtsen, E., editors, *Oil and Gas, Technology and Humans: Assessing the Human Factors of Technological Change*. CRC Press.

Vinnem, J., Bye, R., Gran, B., Kongsvik, T., Nyheim, O., Okstad, E., Seljelid, J., and Vatn, J. (2012). Risk modelling of maintenance work on major process equipment on offshore petroleum installations. *Journal of Loss Prevention in the Process Industries*, 25(2):274–292.