PK8207 - Lecture memo

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Introduction to maintenance optimization

Introduction

The main objective of this course is to increase our understanding of maintenance optimization and improve maintenance modelling skills. With maintenance we understand "the combination of all technical and administrative actions, including supervision actions, intended to retain an item in, or restore to, a state in which it can perform a required function". With maintenance optimisation we understand "balancing the cost and benefit of maintenance". There are many aspects of maintenance optimisation, and some of these are:

- Deciding the amount of preventive maintenance (i.e. choosing maintenance intervals)
- Deciding whether to do first line maintenance (on the cite), or depot maintenance
- Choosing the right number of spare parts in stock
- Preparedness with respect to corrective maintenance
- Time of renewal
- Grouping of maintenance activities.

The main focus in this course will be on optimising preventive maintenance intervals, maintenance limit and time of renewal. Other aspects will however also be treated to some extent.

With preventive maintenance (PM) we understand "the maintenance carried out at predetermined intervals or according to prescribed criteria and intended to reduce the probability of failure or the degradation of the functioning of an item" (EN 13306). There exist several approaches to determine a preventive maintenance program. A concept that is becoming more and more popular is the concept of Reliability Centred Maintenance (RCM). RCM is “a systematic consideration of system functions, the way functions can fail, and a priority based consideration of safety and economics that identifies applicable and effective PM tasks.

An RCM analysis is usually conducted as a pure qualitative analysis with focus on identifying appropriate maintenance tasks. However, the RCM methodology does not give support for quantitative assessment in terms of e.g., interval optimisation. In this course we will present the framework for optimising maintenance interval as well.

The strength of RCM is its systematic approach to consider all system functions, and set up appropriate maintenance task for these functions. On the other hand, RCM is not a methodology that could be used to define a renewal strategy. To determine optimal renewal strategies we will in this course work with Life Cycle Cost modelling (LCC).

The need for programming

In some situations we are able to optimize maintenance without any programming requirements. Simple problems can be solved by Excel, in particular if we have some "easy" built in functions. For many problems we need programming for the optimization. In particular we need programming in order to:

- Model the effect of maintenance on the effective failure rate of components
- Carry out optimization in particular if we have more than one decision variable
- Implement heuristics, in particular when we treat more than one component at a time

We do not recommend a particular programming language. With no programming experience, VBA (Visual Basic for Applications) is recommended because it is not required to install any software to run VBA, and the syntax is very simple. The challenge then is that optimization routines and integration routines have to be programmed.

Experience shows that most students following this course have competence in either Matlab, Python or R. These are all appropriate for this course.

Classical maintenance optimization

Within maintenance optimisation literature it is common to present some basic models such as the Age Replacement Policy (ARP) model, the Block Replacement Model (BRP) and the Minimal Repair Policy (MRP). Such models were introduced by Barlow and Hunter (1960) and have later been generalised in several ways, see e.g. Block et. al. 1988, Aven and Bergman (1986), and Dekker (1992). There exists also several major (review) articles in this area, e.g. Pierskalla and Voelker (1979), Valdez Flores and Feldman (1989), Cho and Parlar (1991) and Wang (2002). (These one were up to 2000..., more later on)

Some of these classical methods will be discussed in this course. However, in order to have a standardized framework for the modelling we will introduce a common term, i.e., the "effective failure rate" which may be applied in very many situations.

The effective failure rate is the expected number of failures per unit time as a function of our preventive maintenance strategy. In the simplest cases the preventive maintenance strategy is to maintain at predefined intervals. We will denote the failure rate by $\lambda_{\mathrm{E}}()$. For example if we as a preventive maintenance activity replace the item at intervals of fixed length τ, we write the effective failure rate as $\lambda_{\mathrm{E}}(\tau)$.

Now there are two challenges, first we want to establish the relation $\lambda_{\mathrm{E}}(\tau)$ depending on the (component) failure model we are working with, then next, we need to specify a cost model to optimise. The cost model will generally involve system models as fault tree analysis, Markov analysis etc. This enables us to find the optimum maintenance intervals in a two step procedure. Note also that when we use $\lambda_{\mathrm{E}}(\tau)$ in the system models we then assume a "constant failure rate" which of course is an approximation for ageing components. However, if the component is maintained preventively it is reasonable that those failures "escaping" our maintenance strategy are independent of time, hence the constant failure rate approximation is reasonable.

Introductory example

Consider a component for which the effective failure rate is given by $\lambda_{\mathrm{E}}(\tau) = \tau/100$, where τ is the maintenance interval. Assume that the cost of a component failure is $C_{\mathrm{F}} = 10$ (corrective maintenance cost, production loss etc). Further let $C_{\mathrm{PM}} = 1$ be the cost per preventive maintenance action carried out at intervals of length τ. The total cost per unit time is then given by:

$$C(\tau) = C_{\mathrm{PM}}/\tau + C_{\mathrm{F}}\tau/100 = 1/\tau + \tau/10 \tag{1}$$

To minimize cost we differentiate, and equate to zero:

$$\frac{dC(\tau)}{d\tau} = \frac{-1}{\tau^2} + \frac{1}{10} = 0 \Rightarrow \tau = \sqrt{10} \approx 3.16 \tag{2}$$

Expanding the cost model

In many situations we would be more explicit on the cost of a failure. A standard form of the cost model to consider is given by:

$$C(\tau) = C_{\mathrm{PM}}/\tau + \lambda_{\mathrm{E}}(\tau)C_{\mathrm{F}} = C_{\mathrm{PM}}/\tau + \lambda_{\mathrm{E}}(\tau)[C_{\mathrm{CM}} + C_{\mathrm{EP}} + C_{\mathrm{ES}} + C_{\mathrm{EM}}] \tag{3}$$

where

- C_{PM} is the cost per preventive maintenance activity
- $C_{\text{CM}}$is the cost of a corrective maintenance activity
- C_{EP} is the expected economic value of production loss upon a failure often expressed as: $C_{\text{EP}} = \Pr(P)[C_{\text{P}}\text{MDT} + C_{\text{T}}]$, where
  - $\Pr(P)$ is the probability that a component failure gives a system failure with production loss
  - C_{P} is the value of production loss per time unit (typically per hour) when the system is down
  - MDT is the mean down time after a failure (typically in hours)
  - C_{T} a fixed cost upon a trip, i.e., when the system goes down independent of the duration of the downtime
- C_{ES} is the expected economic value related to safety loss upon a failure, and is often expressed as: $C_{\text{ES}} = \Pr(S)C_{\text{S}}$, where
  - $\Pr(S)$ is the probability that a component failure gives a system failure with safety impact
  - C_{S} is the corresponding cost given that the "safety event" occurs
- C_{EM} is the expected economical value of material losses upon a component failure.

By an explicit modelling of the failure cost C_{P} we might investigate other aspect of the optimization problem than the effective failure rate. For example MDT might depend on availability of spare parts, preparedness etc, further $\Pr(P)$ might depend on the reliability of backup systems, and $\Pr(S)$ might depend on other safety barriers. In the following we will not pursue this idea, and generally we collect all costs into C_{F}.

The effective failure rate, $\lambda_{\text{E}}()$

There is no general formula for the effective failure. We need to consider each situation individually. However, there are some standard situations where we are able to provide explicit formulas or ways to calculate the effective failure rate.

The "simple" situation

A very simple way to find the effective failure rate is described below. This approach is very often sufficient, although the approximation might be rather rough. Assume that we have an ageing item, i.e., an item with an increasing failure rate function $z(t)$. Further assume that times to failure are Weibull distributed with mean time to failure MTTF and ageing (shape) parameter α. If we replace the item periodically with times between replacements equal to τ, we approximate the effective failure rate with the average failure rate function in the interval $[0,\tau]$. This gives:

$$\lambda_{\mathrm{E}}(\tau) = \left(\frac{\Gamma(1+1/\alpha)}{\mathrm{MTTF}}\right)^{\alpha} \tau^{\alpha-1} \tag{4}$$

A slightly improved approximation

Equation (4) is not very accurate if the $\tau > \mathrm{MTTF}/3$. It might be shown that a better approximation is given by:

$$\lambda_{\mathrm{E}}(\tau) = \left(\frac{\Gamma(1+1/\alpha)}{\mathrm{MTTF}}\right)^{\alpha} \tau^{\alpha-1}\gamma(\tau,\alpha,\mathrm{MTTF}) \tag{5}$$

where the correction term $\gamma(\tau,\alpha,\mathrm{MTTF})$ is given by:

$$\gamma(\tau,\alpha,\mathrm{MTTF}) = \left[1 - \frac{0.1\alpha\tau^2}{\mathrm{MTTF}^2} + \frac{(0.09\alpha-0.2)\tau}{\mathrm{MTTF}}\right] \tag{6}$$

Computationally Equation (5) will not cause any problems. But if we search for analytical solutions we will not be able to find such ones with this improved approximation.

An almost exact approximation

The effective failure rate is the expected number of failures per unit time. Assuming that the item always is replaced by a new one every τ time unit, the expected number of failures in one cycle of length τ is given by the *renewal function*, $W(\tau) = \mathrm{E}(N(\tau))$. This means that the effective failure rate is given by:

$$\lambda_{\mathrm{E}}(\tau) = \frac{W(\tau)}{\tau} \tag{7}$$

From the fundamental renewal equation, $W(t) = F_T(t) + \int_0^t W(t-x)f_T(x)dx$ we are able to set up an iterative scheme to calculate the effective failure rate. Assume we have a reasonable initial approximation, for $W(t)$, say $W_0(t)$. We may then use the following iteration scheme:

$$W_i(t) = F_T(t) + \int_0^t W_{i-1}(t-x)f_T(x)dx \tag{8}$$

to obtain better and better solutions for $W(t)$. An initial approximation would be:

$$W_0(t) = \lambda_{\mathrm{E}}(t)t = \left(\frac{\Gamma(1+1/\alpha)}{\mathrm{MTTF}}\right)^{\alpha} t^{\alpha-1}t = \left(\frac{\Gamma(1+1/\alpha)}{\mathrm{MTTF}}\right)^{\alpha} t^{\alpha} \tag{9}$$

To solve the convolution integral in Equation (8) we need numerical methods. For each iteration we need to maintain a vector of W-values. $F_T(t)$ and $f_T(t)$ are the cumulative distribution function and probability density function for the time to failure respectively, and in our case we usually assume Weibull distributed times to failure. For typical values of $t <$ MTTF the solution converges after 2-3 iterations.

Although we need rather few iterations, computational time might still be long because it is required to calculate the effective failure rate several times in order to minimize total expected cost.

Exercise 1

Consider an old fashioned car with a timing belt. We will decide the optimal interval for replacement of the timing belt.

We assume time-to-failure of the timing belt is Weibull distributed. The following parameters are given:

- MTTF = 175000 km
- $\alpha = 3$ (shape/ageing parameter, medium ageing)
- $C_{\text{PM}} = 7000$ (NOKs)
- $C_{\text{CM}} = 35000$ (NOKs)

a) Find the optimal interval τ by an analytical approach applying Eq. (4) for the effective failure rate

b) Find the optimal interval τ by a numerical approach applying Eq. (5) for the effective failure rate

c) Find the optimal interval τ by a numerical approach applying Eq. (7) for the effective failure rate

d) Make a table where you calculate the effective failure rate for the three models in the interval [0, MTTF]. For which values can we use the simplest formula, for which values can we use the improved version, and for which values is it required to use the approach requiring the renewal function to be calculated?

Exercise 2

We will now also include safety and "unavailability" cost, i.e., we will include the following additional information into the optimization problem:

- Pr(Need to rent a car | Breakdown) = 0.1
- Cost of renting a car = 5000 NOKs

- Pr(Overtaking | Breakdown) = 0.005
- Pr(Collision | Overtaking | Breakdown)=0.2
- Cost of collision = $C_{\text{Collision}}$ = 25 million NOKs

Find the optimal interval with these data.

Exercise 3

Above we introduced the renewal theorem in terms of $W(t) = F_T(t) + \int_0^t W(t-x)f_T(x)dx$ where we assume that all times are identically distributed. The theorem has a more general form

$$W(t) = F_{T_1}(t) + \int_0^t W(t-x)f_T(x)dx \tag{10}$$

where $F_{T_1}(t)$ is the cumulative distribution function for the first time-to-failure, and $f_T(t)$ is the probability density function for all subsequent times-to-failure.

The iterative scheme may in this case be formulated as:

$$W_i(t) = F_{T_1}(t) + \int_0^t W_{i-1}(t-x)f_T(x)dx \tag{11}$$

Assume that the times-to-failure still are Weibull-distributed. Further assume that we know there has been no failures in the time interval $[0, t_0]$.

a) Write a function to calculate $W(t|t_0)$ equal to the expected number of failures in the period $[t_0, t_0 + t]$.

b) Argue why we as an approximation could use:

$$W(t|t_0) \approx (t_0 + t)\lambda_{\text{E}}(t_0 + t) - t_0\lambda_{\text{E}}(t_0) \tag{12}$$

c) Compare the two approaches numerically when $\alpha = 3$, MTTF = 10, $t_0 = 2$ and $t = 3$.