**Problem 1**

Consider a Wiener process with drift *μ* and infinitesimal variance *σ*2.

1. What is the distribution of *Y*(*t*)-*Y*(*s*) provided *t* > *s*?
2. Assume we have a component which deteriorates according to a Wiener process, and the component fails the first time degradation is higher than the value *L*. Let *T* be the lifetime of the component. Why will Pr(*T* ≤ *t*) = Pr(*Y*(*t*)>*L*) not hold in this situation?
3. How can we find Pr(*T* ≤ *t*)?

**Problem 2**

Assume we inspect the component every *τ* unit of times. If the degradation is higher than *y*ML, we replace the unit with a new component. Assume inspection costs is *C*I, renewal cost is *C*R and failure cost is *C*F.

1. Write down the cost equation to minimize. Which quantities do you need?
2. How could you find the effective failure rate?
3. How could you find the renewal rate by a simple argument? Hint: What would be the expected time to the first passage of the maintenance limit?

**Problem 3**

Consider a multistate Markov model with *r*+1 states. Assume that state 0 corresponds to a new component, and state corresponds to a failed component. Assume the only transitions possible in a small time interval is from state *i* to state *i*+1 with transition rate *λ*i. Assume we inspect the component every *τ* unit of times. If the degradation is higher than *l*, we replace the unit with a new component.

1. How could we find the time dependent solution for this model if we do not maintain the component? How would you find MTTF?
2. Using the same cost elements as in Problem 2, how would you optimize the maintenance strategy?
3. What are the arguments for having shorter inspection intervals as the component deteriorate?
4. Explain in brief the main strategies for modelling the system if inspection intervals depend on the state recorded on the last inspection?

**Problem 4**

1. Explain what is meant by dynamic grouping?
2. Let *Kk* be the candidate group with the first *k* “due dates” in a dynamic grouping strategy. Explain each term in the cost equation below:

$$c(t;k)=S+\sum\_{i\in K\_{k}}^{}\left[c\_{i}^{P}+M\_{i}(t-t\_{0}+x\_{i})-M\_{i}(x\_{i})+(T-t)Φ\_{i}^{\*}\right]$$

$$+\sum\_{i\notin K\_{k}}^{}\left[c\_{i}^{P}+S/k\_{i}+M\_{i}(x\_{i}^{\*})-M\_{i}(x\_{i})+(T-t\_{i}^{\*})Φ\_{i}^{\*}\right]$$

1. Explain how to obtain an optimal grouping strategy.