**Exam TPK4140 – Autumn 2021**

We shall evaluate the maintenance of a pump system. The pump system comprises two pumps, an active pump (A) and a stand-by pump (B) as shown in Figure 1.



**Figure 1 Pump system**

If the active pump (A) fails, an attempt is made to start the stand-by pump (B). The switch-over time before the stand-by pump will be operative can be ignored. Further the stand-by pump might have failed in stand-by mode. We assume that the active pump has an increasing failure rate function, *z*(*t*), whereas the stand-by pump has a constant failure rate function (in stand-by mode). We assume that the stand-by pump will run for a short period of time upon failure of the active pump. In this period we assume that the probability of failure of the stand-by pump equals zero.

Preventive maintenance of the active pump is periodical replacement of wear parts. The maintenance of the stand-by pump is a periodical functional test to reveal any hidden failures in stand-by position. The relevant quantities to consider in the optimization are as follows.

| Parameter | Value | Explanation |
| --- | --- | --- |
| MTTF*A* | 8 760 | Mean time to failure for pump A, in hours (without preventive maintenance) |
| MTTF*B* | 17 520 | Mean time to failure for pump B, in hours. The MTTF*B* is related to the stand-by mode of operation, we ignore the probability of failure if the stand-by pump is in an operative mode. |
| α | 3 | Aging parameter for pump A, assuming Weibull distributed time-to-failures  |
| CPM | 3 000 | Cost of execution of a PM activity for pump A |
| CPT | 1 000 | Cost of execution of a proof-test of pump B |
| CCM | 10 000 | Cost of repairing a pump upon failure. Same cost for both pumps |
| MDT | 4 | Mean down time in case of stand-by pump will not start. Downtime is assumed to be exponentially distributed |
| *CU* | 15 000 | Cost of one hour loss of production |
| *C*Trip | 100 000 | Trip cost, i.e., upon a failure of both pumps |
| *τA* | 2 190 | Maintenance interval for the active pump, pump A (initial maintenance program) |
| *τB* | 8 760 | Proof-test interval for stand-by pump, pump B (initial maintenance program) |

In this problem you may use

* The effective failure rate for the active pump is given by: $λ\_{E}\left(τ\right)=\left(\frac{Γ\left(1+\frac{1}{α}\right)}{MTTF}\right)^{α}τ^{α-1}$
* Γ(1+1/3) = 0.893
* For a stand-by unit which is periodically tested at intervals of length *τ*, the probability of failure on demand is given by PFD = *λτ*/2 = *τ*/(2MTTF)
1. Write down an expression for the expected cost per unit time as a function of *τA* and *τB*. Assume that we fix *τB* = 8 760. Find an analytical expression for the optimal interval for the active pump, *τA*. Insert values to find a numerical answer for the optimal interval.
2. Given that we fix *τA* to the value found in problem a), what is then the optimal proof-test interval, *τB*, for pump B.
3. Calculate the total expected cost with the initial intervals, with the intervals used in a), and the final intervals obtained in b).
4. Explain important steps in an RCM analysis and the use of the RCM-decision logic. Give examples related to the pump system.

Solution:

$$C\left(τ\_{A},τ\_{B}\right)=\frac{C\_{PM}}{τ\_{A}}+\frac{C\_{PT}}{τ\_{B}}+λ\_{E}\left(τ\_{A}\right)\left[C\_{CM}+\frac{τ\_{B}}{2MTTF\_{B}}\left(C\_{U}MDT+C\_{Trip}\right)\right]+\frac{C\_{CM}}{MTTF\_{B}+\frac{τ\_{B}}{2}}$$

We use:$\frac{C\_{CM}}{MTTF\_{B}+\frac{τ\_{B}}{2}}=\frac{C\_{CM}}{MTTF\_{B}(1+\frac{τ\_{B}}{2MTTF\_{B}})}≈\frac{C\_{CM}}{MTTF\_{B}}(1-\frac{τ\_{B}}{2MTTF\_{B}})$

Hence, to minimize with respect to proof test we use $∂$

$$C\left(τ\_{A},τ\_{B}\right)=\frac{C\_{PM}}{τ\_{A}}+\frac{C\_{PT}}{τ\_{B}}+λ\_{E}\left(τ\_{A}\right)\left[C\_{CM}+\frac{τ\_{B}}{2MTTF\_{B}}\left(C\_{U}MDT+C\_{Trip}\right)\right]+\frac{C\_{CM}}{MTTF\_{B}}(1-\frac{τ\_{B}}{2MTTF\_{B}})$$

and we seek

$$\frac{∂C\left(τ\_{A},τ\_{B}\right)}{∂τ\_{B}}=-\frac{C\_{PT}}{τ\_{B}^{2}}+λ\_{E}\left(τ\_{A}\right)\left[\frac{C\_{U}MDT+C\_{Trip}}{2MTTF\_{B}}\right]-\frac{C\_{CM}}{2MTTF\_{B}^{2}}=0$$

Yielding

$$τ\_{B}^{\*}=\sqrt{\frac{2C\_{PT}MTTF\_{B}}{λ\_{E}\left(τ\_{A}\right)\left[C\_{U}MDT+C\_{Trip}\right]-C\_{CM}/MTTF\_{B}}}$$

Remaining part of solution in Excel file.