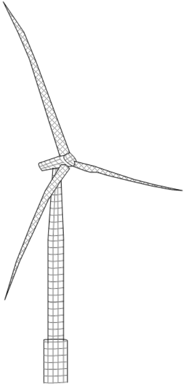
NTNU | Norwegian University of Science and Technology

DIGITAL TWIN - MAINTENANCE AND OPERATIONS

Jørn Vatn/November-2025

Digital twins

Digital twins



Introduction

- A Digital Twin (DT) in the current context is aimed as a *virtual representation* of a technical system, a production plant etc.
- Key elements of a DT are bigdata, artificial intelligence and simulations for real-time prediction, monitoring, control and optimization
- The DNV-RP-A204 Qualification and assurance of digital twins classifies level of capability of Digital Twin on a scale from 0-5 (0-standalone, 1-descriptive, 2-diagnostic, 3-predictive, 4-prescriptive, 5-autonomy)
- The objective of this lecture is to give some ideas related to level 3 and 4 on the DNV ladder

DNV-RP-A204

DNV

RECOMMENDED PRACTICE

DNV-RP-A204

Edition October 2020
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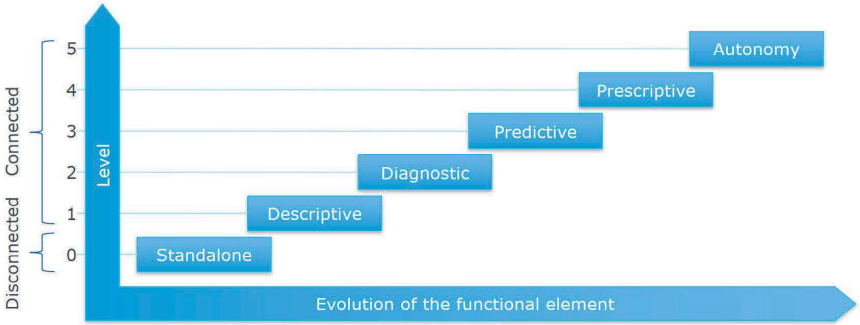
Qualification and assurance of digital twins

DNV-RP-A204 - Evolution stages or capability of a functional element



Introduction

- We will build on the maintenance example from the reliability and maintenance lecture
- Details regarding the models are found in the course compendium
- One additional model is introduced
- Some of the maintenance concepts have similarities with the *inventory models*

Introduction

The digital twins (DT) are labelled as follows:

- The *Operations* DT: This DT contains operational plans, cost related to lost production, opportunity windows for maintenance etc. Example: Automatic Stock Replenishment Software, EOQ, (r-Q) etc
- The *Condition* DT: This DT contains information regarding the condition of the "hardware". In principle this DT contains both current condition, and historical data. CMMS, SCADA, cond.monitoring system
- The *Maintenance cost* DT: This DT contains the relevant mathematical models used for optimizing maintenance, interacting with the Operations DT

We will not elaborate on technical issues, i.e., how these DT are sharing data, how we can conduct "what-if" queries etc.

Single component - Maintenance cost digital twin

Repetition:

- A component has an increasing failure rate function, $z(t)$
- At predetermined point of times, $\tau, 2\tau, 3\tau, \ldots$, the component is preventively restored to an *as good as new* condition
- The average cost per unit time is given by:

$$C(\tau) = c_{\mathrm{PM}}/\tau + \lambda_{\mathrm{E}}(\tau) c_{\mathrm{U}}$$

where

  - c_{PM} is the cost of a preventive maintenance action
  - $\lambda_{\mathrm{E}}(\tau)$ is the *effective failure rate*
  - c_U = unplanned cost given that a failure occurs

Single component - Condition digital twin

- Actually, we do not have a Condition DT in the example
- Historical data could in principle be available, i.e., information regarding failure times etc.
- Data analytics: The DT could in principle conduct statistical analysis on the data, to provide MTTF and other parameters, see chapter on parameter estimation

Single component - Operations digital twin

- Also here, we do not have an Operations DT
- Indirectly we have an operations DT in terms of c_{PM} and c_{U} depending on how production is affected by maintenance and failures

Effective failure rate, $\lambda_{\mathrm{E}}(\tau)$

- In the case of Weibull distributed times to failure we may find approximation formulas for the effective failure rate:
- Let MTTF be the mean time to failure without maintenance, and α be the ageing parameter of the lifetime distribution of the component
- An approximation of the effective failure rate is:

$$\lambda_{\mathrm{E}}(\tau) = \left(\frac{\Gamma(1 + 1/\alpha)}{\mathrm{MTTF}}\right)^{\alpha} \tau^{\alpha-1}$$

- Optimal value of τ is given by:

$$\tau^{*} = \frac{\mathrm{MTTF}}{\Gamma(1 + 1/\alpha)} \left(\frac{c_{\mathrm{PM}}}{c_{\mathrm{U}}(\alpha - 1)}\right)^{1/\alpha}$$

Example

- In a production line a packing machine is critical for the production
- $\mathrm{MTTF} = 600$ hours ($\approx$ one month) if the machine is not preventively maintained
- Failure times are assumed to be Weibull distributed with ageing parameter $\alpha = 4$
- The cost of a preventive maintenance activity is $c_{\mathrm{PM}} = 5\,000$ NOKs
- If the machine fails the total cost of corrective maintenance and lost production is $c_{\mathrm{U}} = 35\,000$
- Assume "24/7"

$$\tau^* = \frac{\mathrm{MTTF}}{\Gamma(1 + 1/\alpha)} \left(\frac{c_{\mathrm{PM}}}{c_{\mathrm{U}}(\alpha - 1)} \right)^{1/\alpha} \approx \frac{600}{0.906} \left(\frac{5000}{35000 \times 3} \right)^{1/4} \approx 309 \text{ hours}$$

▸ Show in Excel

Example 1, synchronization

- 309 hours $\approx$ 1.8 weeks
- From the operations DT: We offer maintenance windows only every Monday morning
- What to do?

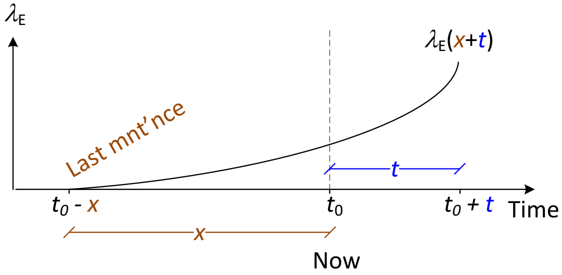
▸ Show in Excel

Example 2, synchronization

- Assume maintenance every second week, $\tau^* = 14$ days
- It is due time to do the schedule maintenance, but production demand requires to skip the maintenance, and wait one week more
- What are the pros and cons of postponing maintenance?

Example 2, synchronization, cont.

- $C(\tau^*) = c_{\mathrm{PM}}/\tau^* + \lambda_{\mathrm{E}}(\tau^*)c_{\mathrm{U}} \approx$ 3 660 NOKs per week
- What is the cost of deferred maintenance?
- Assume that current time is t_0 and x time units has elapsed since last preventive maintenance
- We consider to run the system for another t time units before the next PM activity is performed



Example 2, synchronization, cont.

- The total unplanned failure cost in this entire time period from $t_0 - x$ to $t_0 + t$ is:

$$C_U(0, x+t) = \lambda_{\mathrm{E}}(x+t) \cdot (x+t) \cdot c_{\mathrm{U}}$$

- We have already "paid" unplanned cost from $t_0 - x$ to t_0 equal to $C_U(0, x) = \lambda_{\mathrm{E}}(x) \cdot x \cdot c_{\mathrm{U}}$, hence the cost in the coming t time units will be:

$$C_U(x, x+t) = C_U(0, x+t) - C_U(0, x)$$
$$= \lambda_{\mathrm{E}}(x+t) \cdot (x+t) \cdot c_{\mathrm{U}} - \lambda_{\mathrm{E}}(x) \cdot x \cdot c_{\mathrm{U}}$$

Example 2, synchronization, cont.

- In the example x = 2 weeks = 336 hours and t = 1 week = 168 hours yielding

$$
\begin{aligned}
C_U(x = 336, x + t = 336 + 168) &= C_U(0, 504) - C_U(0, 336) \\
&= \lambda_{\mathrm{E}}(504) \cdot 504 \cdot c_{\mathrm{U}} - \lambda_{\mathrm{E}}(336) \cdot 336 \cdot c_{\mathrm{U}} \\
&\approx 11\,760 - 2\,320 = 9\,438 \text{ NOKs}
\end{aligned}
$$

- The cost of 9 438 NOKs should then be compared with the average cost per week $\approx$ 3 660 NOK
- The increase in cost is almost 6 000 NOKs and this amount should then be compared to the value of not interrupting the production with maintenance, i.e., information from the operations DT

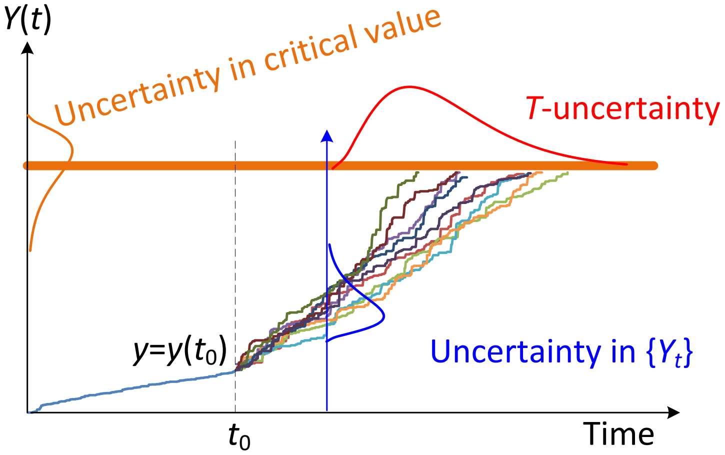
Predictive maintenance - PdM

- In contrast to traditional calendar based preventive maintenance the main idea of a predictive maintenance strategy is to utilize component condition, future loads, and opportunity windows to determine a “just in time” plan for maintenance
- Condition information is basically used for:
  1. Anomaly detection, i.e., early warning of coming events
  2. Diagnostics, i.e., the search for root causes behind symptoms observed
  3. Prognostics, i.e., estimation of degradation rate, time to failure, remaining useful life etc. based on relevant information
- In this lecture we only focus on prognostics and decision making

Model 1 - Condition digital twin

- At time t_0 we observe the state/condition of a component in terms of a state vector $\mathbf{y}$
- Let t denote running time from t_0
- Let T denote the time-to-failure of the component from t_0
- Given $\mathbf{y}$ assume that it is possible to describe the probability distribution of T, say $T \sim \text{Weibul}(\alpha, \lambda)$

Model 1 - Condition digital twin



Note

In the following we distinguish between:

- $\{X(t), t \geq 0\}$ is a stochastic process describing the actual degradation of the item at time t
- $\{Y(t), t \geq 0\}$ is a stochastic process describing the *measurements* of degradation of the item at time t

Note

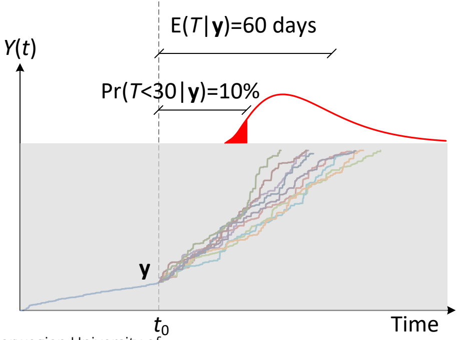
For example:

- $\{X(t), t \geq 0\}$ wear level of outer bearing ring at time t
- $\{Y(t), t \geq 0\}$ A FFT-measurement at time t

$\{Y(t), t \geq 0\}$ may have large local variation due to variation in wind speed, whereas $\{X(t), t \geq 0\}$ is almost not changing

Model 1 - Condition digital twin

- Given **y** assume that it is possible to describe the probability distribution of T, say $T \sim \text{Weibul}(\alpha, \lambda)$
- An experienced maintenance engineer suggests:
  - The mean time to failure, i.e., $\text{E}(T|\mathbf{y})$ = 2 months = 60 days, and
  - The probability that the component will fail within one month = 10%



Model 1 - Operations digital twin

- "Maintenance windows" available every $\tau = 7$ days
- The first one will come in $\tau_1 = 3$ days, the next in $\tau_1 + \tau = 10$ days etc
- Cost of utilizing an opportunity $c_{\mathrm{PM}}(t) = c_{\mathrm{PM},0} e^{-t/\theta}$
  - where $\theta = 30$ days is a characteristic time for the decrease in the sense that for $t = \theta$ the cost has dropped to $e^{-1} \approx 37\%$

Model 1 - Maintenance cost digital twin

- Objective function:

$$C(t) = c_{\mathrm{PM},0} e^{-t/\theta} + c_{\mathrm{U}} F_T(t)$$

- $c_{\mathrm{PM},0}$ = 10 000 NOKs
- c_{U} = 35 000 NOKs = cost if failure
- $T \sim$ Weibull, where
  - $\mathrm{E}(T) = \Gamma(1 + 1/\alpha)/\lambda = 60$ days
  - $F_T(t) = 1 - e^{-(\lambda t)^\alpha}$, and $F_T(30 \text{ days}) = 10\%$
- We find, see compendium, $\alpha \approx 2.78$ by a fix-point iteration scheme, and then $\lambda \approx 0.0148$

Model 1 - Solution

t	PM	U	Total
3	9 048	6	9 054
10	7 165	173	7 339
17	5 674	752	**6 426**
24	4 493	1 928	**6 421**
31	3 558	3 815	7 373

This means that the optimal time for performing the PM task will be the third or forth opportunity

Model 2 - Condition digital twin, more detailed on “condition”

- We assume that an “anomaly” has been detected
- Time-to-failure is still Weibull distributed, i.e., $z(t) = \alpha\lambda^{\alpha} t^{\alpha-1}$
- The so-called Cox-proportional hazard model is often used to incorporate the current state in the failure rate function
  - Let $\mathbf{y}$ be the vector of current relevant state/condition information for the component, for example temperature, vibration level and so on
  - Let $\overline{\mathbf{x}(t)}$ be the vector of future average loads in the time period $[0, t)$
- The failure rate function may be written on the form:

$$z(t|\mathbf{y}, \overline{\mathbf{x}(t)}) = z_0(t) e^{\beta_{\mathrm{Y}}\mathbf{y}} e^{\beta_{\mathrm{X}}\overline{\mathbf{x}(t)}} = \alpha\lambda^{\alpha} t^{\alpha-1} e^{\beta_{\mathrm{Y}}\mathbf{y}} e^{\beta_{\mathrm{X}}\overline{\mathbf{x}(t)}}$$

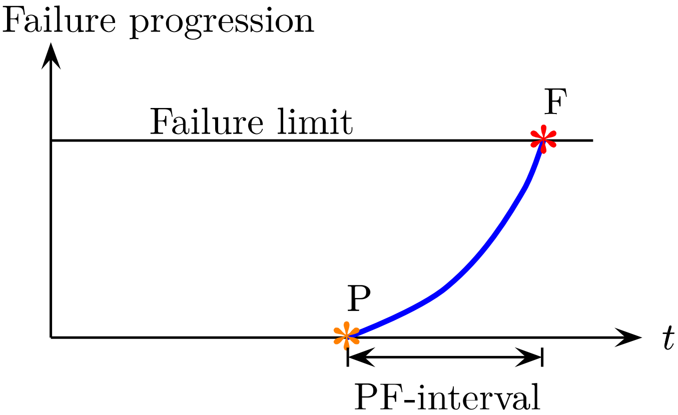
where $\boldsymbol{\beta}_{\mathrm{Y}}$ and $\boldsymbol{\beta}_{\mathrm{X}}$ are regression coefficient vectors

Model 2 - Condition digital twin, cont

- Assume that the parameters $\alpha, \lambda, \boldsymbol{\beta}_{\mathrm{Y}}$ and $\boldsymbol{\beta}_{\mathrm{X}}$ are estimated based on historical data and/or use of expert judgement
- Assume that the current component state/condition, $\mathbf{y}$ is observed by use of sensor technology and/or other inspection methods
- The future load $\overline{\mathbf{x}(t)}$ is obtained from the "operations twin", either for a fixed value, or for different scenarios
- The cumulative distribution function is given by:

$$F_T(t|\mathbf{y}, \overline{\mathbf{x}(t)}) = 1 - \exp\left(-\int_0^t z(u|\mathbf{y}, \overline{\mathbf{x}(u)})\, du\right)$$

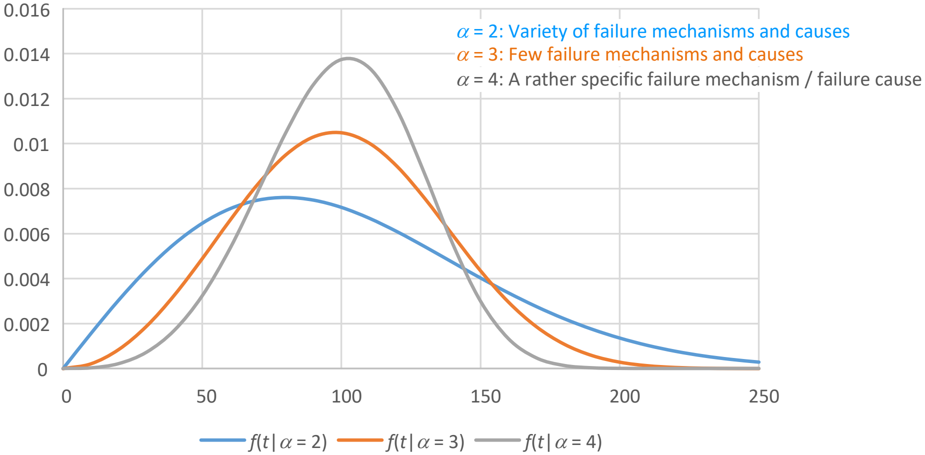
Model 2 - Condition digital twin: PF-model



Model 2 - Condition digital twin / Expert judgement

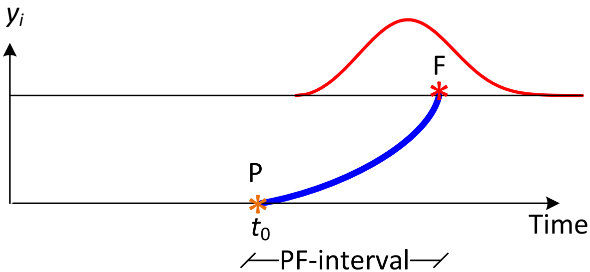
- The PF-interval is a random quantity, and we may use expert judgement for elicitation of the model parameters, α, λ, $\boldsymbol{\beta}_{\mathrm{Y}}$ and $\boldsymbol{\beta}_{\mathrm{X}}$
- The procedure is now as follows
- *Step 1:* Assess ξ = *expected* length of the PF-interval under the assumption of insignificant future load $\overline{\mathbf{x}(t)}$
- *Step 2:* Asses the consistency of the PF-interval by the shape parameter α in the Weibull distribution. As a rule of thumb use:
  - $\alpha = 2$ corresponds to a variety of failure mechanisms and causes behind the failure progression
  - $\alpha = 3$ corresponds to a few failure mechanisms and causes behind the failure progression
  - $\alpha = 4$ corresponds to a rather specific failure mechanism / failure cause behind the failure progression

Model 2 - The shape parameter α ($\xi = 100$)



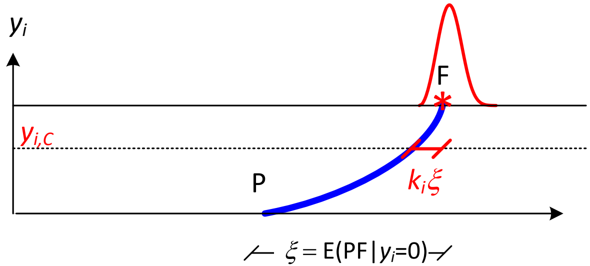
Model 2 - Condition digital twin / Expert judgement

- *Step 3:* Calculate the intensity parameter by $\lambda = \Gamma\left(1/\alpha + 1\right)/\xi$
- For each y_i in **y** let $y_i = 0$ correspond to the condition at the point t_0 in the PF-model. This corresponds to *no significant degradation* for the actual regression variable.



Model 2 - Condition digital twin / Expert judgement

- *Step 4:* For each y_i in $\mathbf{y}$ let $y_{i,\mathrm{C}}$ be a critical value for that particular regression variable. Under the assumption that all other regression variables $y_j = 0, j \neq i$ assess the reduction in ξ by some factor, say k_i.



- Note that there is no specific "rule" to determine $y_{i,\mathrm{C}}$, and the higher value chosen, the lower value will be assessed for k_i

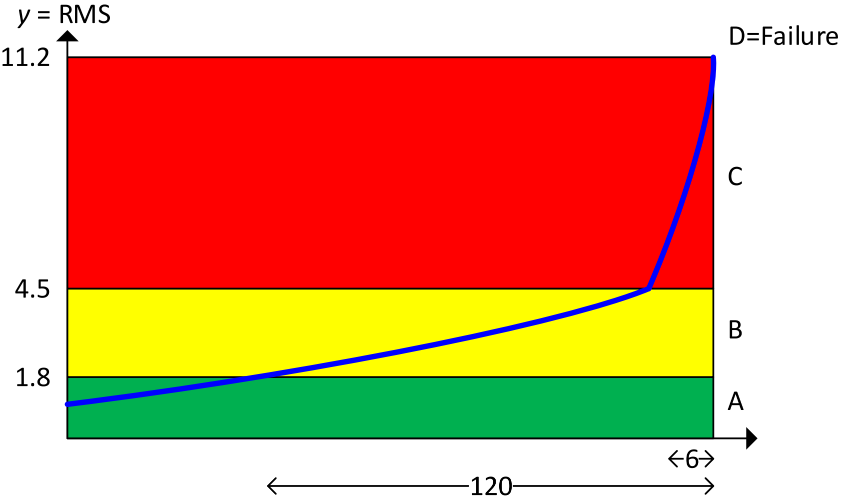
Model 2 - Condition digital twin / Expert judgement

- *Step 5:* Calculate the corresponding regression parameters by $\beta_i = -(\ln k_i)/y_{i,\mathrm{C}}$, i.e., for the elements in $\boldsymbol{\beta}_{\mathrm{Y}}$
- *Step 6:* Repeat the procedure for each $\overline{x_i(t)}$ in $\overline{\mathbf{x}(t)}$ to find the elements of $\boldsymbol{\beta}_{\mathrm{X}}$

Model 2 - Using vibration data and expected loads

- Let y be the vibration level measured by the so-called “RMS" value (Root Mean Square) which is an ISO convention
- For machines of medium size the vibration level is mapped into zones
  - Zone A is the normal level which we here assume corresponds to $y = 0$
  - Zone B which still is considered acceptable ranges from $y = 1.8$ to $y = 4.5$
  - Zone C which is critical ranges from $y = 4.5$ to $y = 11.2$
  - Zone D corresponds to $y > 11.2$
- A machine in zone D is considered to have serious damages within very short time and is therefore often protected by a protection system causing the machine to shut down (TRIP)
- Let x = the portion of time the machine is run on more than 90% of maximum capacity

Model 2 - Using vibration data and expected loads



Model 2 - Elicitation of model parameters, α, λ

- *Step 1:* The maintenance engineer assess the time until the protection system will trip the system to be ξ = 120 days when an anomaly situation occur, i.e., drifting into zone B
- *Step 2:* Since only vibration and excessive load are considered as influencing factors of the PF-interval the shape parameter is assessed by $\alpha = 4$
- *Step 3:* $\implies \lambda = \Gamma\left(1/\alpha + 1\right)/\xi = \Gamma(1.25)/120 \approx 0.00775.$

Model 2 - Elicitation of model parameters, $\beta_{\mathrm{Y}}, \beta_{\mathrm{X}}$

- *Step 4:* The critical vibration level = $y_{\mathrm{C}} = (4.5 - 1.8) = 2.7$ relative to $y = 0$, and the corresponding reduction factor for the remaining time to trip is assessed by the maintenance engineer to be $k_{\mathrm{Y}} = 0.05$ (only 6 days to failure in average)
- *Step 5:* $\implies \beta_{\mathrm{Y}} = -(\ln k_{\mathrm{Y}})/y_{\mathrm{C}} = -(\ln 0.05)/4.5 \approx 1.11$
- *Step 6:* A machine running with 90% of maximum capacity or more in $x_{\mathrm{C}} = 0.25 = 25\%$ of the time is assessed to have a reduction factor of $k_{\mathrm{X}} = 0.1$ (12 days to failure in average)
- $\implies \beta_{\mathrm{X}} = -(\ln k_{\mathrm{X}})/x_{\mathrm{C}} = -(\ln 0.1)/.25 \approx 9.2$

Note since $y = 1.8 \neq 0$ for the baseline case, we subtract 1.8 in the procedure to establish β_{Y}, and we then need to use $y - 1.8$ in the Cox model

Model 2 - Maintenance cost digital twin

- $C(t, x) = c_{\mathrm{PM},0} e^{-t/\theta} + c_{\mathrm{U}} F_T(t|y, x) + p_{\mathrm{C}}(1 - x)$
- $F_T(t|x, y) = 1 - \exp\left(-\int_0^t z(u|y, x)\, du\right)$
- $z(t|y, x) = \alpha\lambda^{\alpha} t^{\alpha-1} e^{\beta_{\mathrm{Y}}(y-1.8)+\beta_{\mathrm{X}} x}$
- $c_{\mathrm{PM},0} = 15\,000$ NOKs
- $c_{\mathrm{U}} = 35\,000$ NOKs = cost if failure
- $p_{\mathrm{C}} = 5\,000$ NOKs = extra daily profit running “full speed”
- $t \in \{t_1, t_1 + \tau, t_2 + \tau, \ldots\}$
- From the condition DT model: $y = 3$
- From the operations DT model: $x = 0.4$

Show in Excel

Brownian motion and the Wiener process

Brownian motion is the random motion of particles in a fluid where collisions between particles result in a rather chaotic behaviour. Brownian motion is described by a continuous-time stochastic process named the Wiener process, i.e., $\{W(t), t \in \Theta\}$ is characterized by:

- $W(0) = 0$
- $\{W(t)\}$ is almost surely continuous
- $\{W(t)\}$ has independent increments
- $W(t) - W(s) \sim \mathcal{N}(0, t - s)$ (for $0 \leq s \leq t$)

where $\mathcal{N}(\mu, \sigma^2)$ denotes the normal distribution with expected value μ and variance σ^2

Wiener process

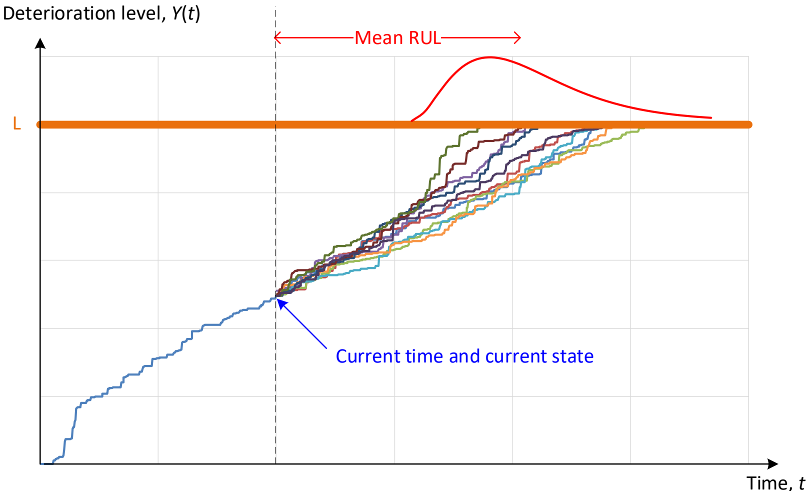
The Wiener process defined above is fluctuating around zero. A related stochastic process is defined by:

$$Y(t) = \mu t + \sigma W(t)$$

This process is called a Wiener process with drift μ and infinitesimal variance σ^2. For the Wiener process with drift we have:

$$Y(t) - Y(s) \sim \mathcal{N}(\mu(t-s), \sigma^2(t-s))$$

Wiener process and time to failure



Wiener process and time to failure

- Let $\{Y(t)\}$ be a stochastic process describing the degradation of a component
- Assume the component will fail the first time $Y(t) > L$
- Let T be the time to failure for the component
- Since the increments could be negative, the relation $F_T(t) = \Pr(T \leq t) = \Pr(Y(t) \geq L)$ *does not hold* for the Wiener process
- It may be shown that the time to failure, i.e., the time to the first passage of the limit L is inverse-Gauss distributed:

Wiener process and time to failure

$$F_T(t) = \Phi\left(\frac{\sqrt{\lambda}}{\nu}\sqrt{t} - \sqrt{\lambda}\frac{1}{\sqrt{t}}\right) + \Phi\left(-\frac{\sqrt{\lambda}}{\nu}\sqrt{t} - \sqrt{\lambda}\frac{1}{\sqrt{t}}\right) e^{2\lambda/\nu}$$

and

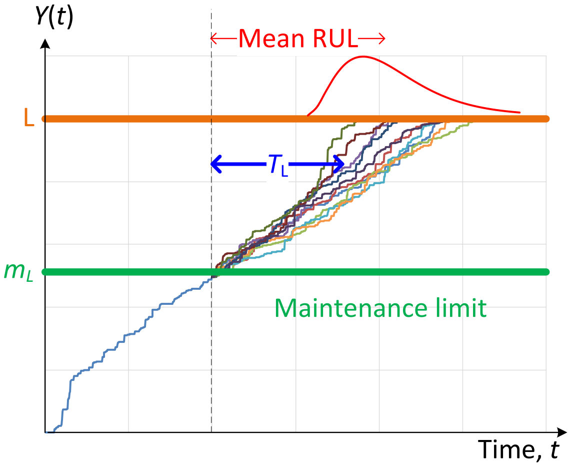
$$\begin{aligned} \mathrm{E}(T) &= \nu \\ \mathrm{Var}(T) &= \nu^3/\lambda \end{aligned}$$

where $\nu = L/\mu$ and $\lambda = L^2/\sigma^2$. In terms of the original parameters we have $\mathrm{E}(T) = L/\mu$ and $\mathrm{Var}(T) = L\sigma^2/\mu^3$.

Wiener process and maintenance cost digital twin

- Assume that we can observe the degradation process continuously
- When degradation *approaches* the failure limit, L, we will place a request to replace the component with a new component
- We assume that there is a deterministic lead time, say T_{L}
- The objective is to determine the maintenance limit, m_{L}, i.e., how close to the failure limit we dear to go

Wiener process with maintenance and failure limit



Wiener process and maintenance cost digital twin

The cost equation to minimize is:

$$C(m_L) = \frac{c_R + c_F F(T_L|m_L) + c_U \int_0^{T_L} f(t|m_L)(T_L - t)dt}{\mathrm{MTBR}(m_L)}$$

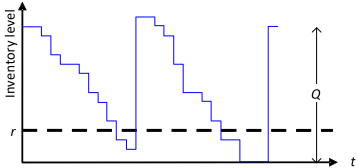
where

- c_R = cost of renewal/replacement
- $c_F = c_{CM} + c_T$ = cost of failure (additional cost for corrective maintenance (CM) and cost for the failure event (T=Trip))
- c_U = cost per hour down time
- $F()$ and $f()$ are CDF and PDF for remaining life time given maintenance limit m_L
- $\mathrm{MTBR}(m_L) = m_L/\mu + T_L$ = Mean Time Between Renewals

Show in Excel (Average)

Similarities with the (r, Q) inventory model

- The lot size, reorder point policy; (r, Q) is visualized by:



- The drop in the inventory level corresponds to the increase in the degradation level for the maintenance model
- The reorder point, r, corresponds to the maintenance limit m_{L}
- In the maintenance model we only decide to “order a renewal”, whereas in the inventory model we also consider how many items to order, Q
- Crossing the failure limit L in the maintenance model corresponds to a stock-out situation in the inventory model

Seasonal differences

- In the Excel model we assumed a set of average values for many model parameters
- In reality these are changing almost hour from hour
- Digital twins will be required for
  - Maintenance cost
  - Operations, including loss production
  - Weather if this influences e.g., lead times (offshore wind)
  - Condition
- The model distinguishes between “summer” and “winter” as a first “real-time DT”

▸ Show in Excel (Winter & Summer)

Relax cost

- When we reach the maintenance limit we could relax on production, e.g.,:
  - Produce less items
  - Stop a wind turbine when wind speed $> 15m/s$
- Let x be a measure of how much we relax on production, i.e., a decision variable
- Let $c_{\mathrm{Rx}}(x)$ be corresponding production loss per unit time
- The cost equation to minimize now is:

$$C(m_{\mathrm{L}}, x) = \frac{c_{\mathrm{R}} + c_{\mathrm{F}} F(T_{\mathrm{L}} | m_{\mathrm{L}}, x) + c_{\mathrm{U}} \int_0^{T_{\mathrm{L}}} f(t | m_{\mathrm{L}}, x)(T_{\mathrm{L}} - t)dt + c_{\mathrm{Rx}}(x) L_{\mathrm{T}}}{\mathrm{MTBR}(m_{\mathrm{L}})}$$

where $F()$ and $f()$ now depends on x as well as the maintenance limit

Show in Excel

Parameter estimation - Condition DT and data analytics

- A digital twin should be able to update it's self
- Here: Estimate μ and σ on the fly
- Assume we are observing in real time the process on a daily basis, i.e., we have observations $y_0, y_1, y_2, \ldots$
- Since the increments are normally distributed, we calculate $x_i = y_i - y_{i-1}$
- Standard estimators for μ and σ^2 are:

$$\hat{\mu} = \overline{x} = 1/n \sum_{i=1}^{n} x_i$$

$$\hat{\sigma}^2 = 1/(n-1) \sum_{i=1}^{n} (x_i - \overline{x})^2$$

Exponential smoothing

- There might be arguments supporting that the degradation process is not stable, it might be variations due to season, operational conditions etc.
- Therefore we will not put so much confident in early observations
- *Exponential smoothing* is a technique where we give higher weight to the latest observations
- A smoothing parameter, say α is introduced in exponential smoothing:

$$\hat{\mu}_n = \alpha\left(x_n + (1-\alpha)x_{n-1} + (1-\alpha)^2 x_{n-2} + \cdots + (1-\alpha)^{n-1}x_1\right)$$

- For example, if $\alpha = 0.1$ we put 10% weight of the first observation, 9% weight on the second and so on, and the result will be that early observations are wiped out
- It is common to use $\hat{\sigma}^2 = 1/(n-1)\sum_{i=1}^{n}(x_i - \overline{x})^2$ if we need the variance even if we do exponential smoothing

Bayesian updating

- Assume we have a new component where we do not know μ and σ
- Assume we have observed similar components, and over time we have estimated μ_i and σ_i, $i = 1, 2, \ldots, m$
- To simplify, we assume that σ is rather consistent, i.e., not much variance, and therefore we assume σ is "almost" known
- We observe some variation in μ, i.e., some components degrade faster than others
- The information from these other components is the basis for our *prior belief* about the unknown parameter μ:
  - $\mu_0 = \frac{1}{m} \sum_i \mu_i$ = Mean drift in the sample of components
  - $\tau_0^2 = \frac{1}{m-1} \sum_1 (\mu_i - \mu_0)^2$ = Sample variance in the drift parameter

Bayesian updating

- The Bayesian posterior mean and variance for μ is given by:

$$
\begin{aligned}
\hat{\mu}_{\mathrm{B}} &= \left(\frac{1}{\tau_n^2} + \frac{1}{\tau_0^2}\right)^{-1} \left[\frac{\hat{\mu}_n}{\tau_n^2} + \frac{\mu_0}{\tau_0^2}\right] \\
\hat{\tau}_{\mathrm{B}}^2 &= \left(\frac{1}{\tau_n^2} + \frac{1}{\tau_0^2}\right)^{-1}
\end{aligned}
$$

- where

$$
\begin{aligned}
\hat{\mu}_n &= \alpha\left(x_n + (1-\alpha)x_{n-1} + (1-\alpha)^2 x_{n-2} + \cdots + (1-\alpha)^{n-1} x_1\right) \\
\tau_n^2 &= \sigma^2 \frac{\alpha - \alpha^2 + 2(1-\alpha)^{2n}}{(\alpha-2)(\alpha-1)}
\end{aligned}
$$

Maintenance cost digital twin

- Recall the cost equation:

$$C(m_{\mathrm{L}}, x) = \frac{c_{\mathrm{R}} + c_{\mathrm{F}} F(T_{\mathrm{L}} | m_{\mathrm{L}}, x) + c_{\mathrm{U}} \int_0^{T_{\mathrm{L}}} f(t | m_{\mathrm{L}}, x)(T_{\mathrm{L}} - t) dt + c_{\mathrm{Rx}}(x) L_{\mathrm{T}}}{\mathrm{MTBR}(m_{\mathrm{L}})}$$

- where both $F()$ and $f()$ implicitly depend on μ and σ
- We assume that σ is known, but μ is unknown
- We continue to update $\hat{\mu}_{\mathrm{B}}$ and $\hat{\tau}_{\mathrm{B}}^2$ as we get new observations
- To calculate $F()$ and $f()$ with the latest information regarding μ we need to "integrate" over the uncertainty distribution of μ , i.e., $\hat{\mu}_{\mathrm{B}}$ and $\hat{\tau}_{\mathrm{B}}^2$ (law of total probability)
- We may discretize as we did for stochastic programming

Thank you for your attention

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