NTNU | Production Management Research Group

- LOGISTICS 4.0 LAB -

ASSEMBLY SYSTEMS DESIGN AND BALANCING

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Agenda

1. Introduction to the lecture
2. Design and Balancing mathematical method (Patterson & Albracht)

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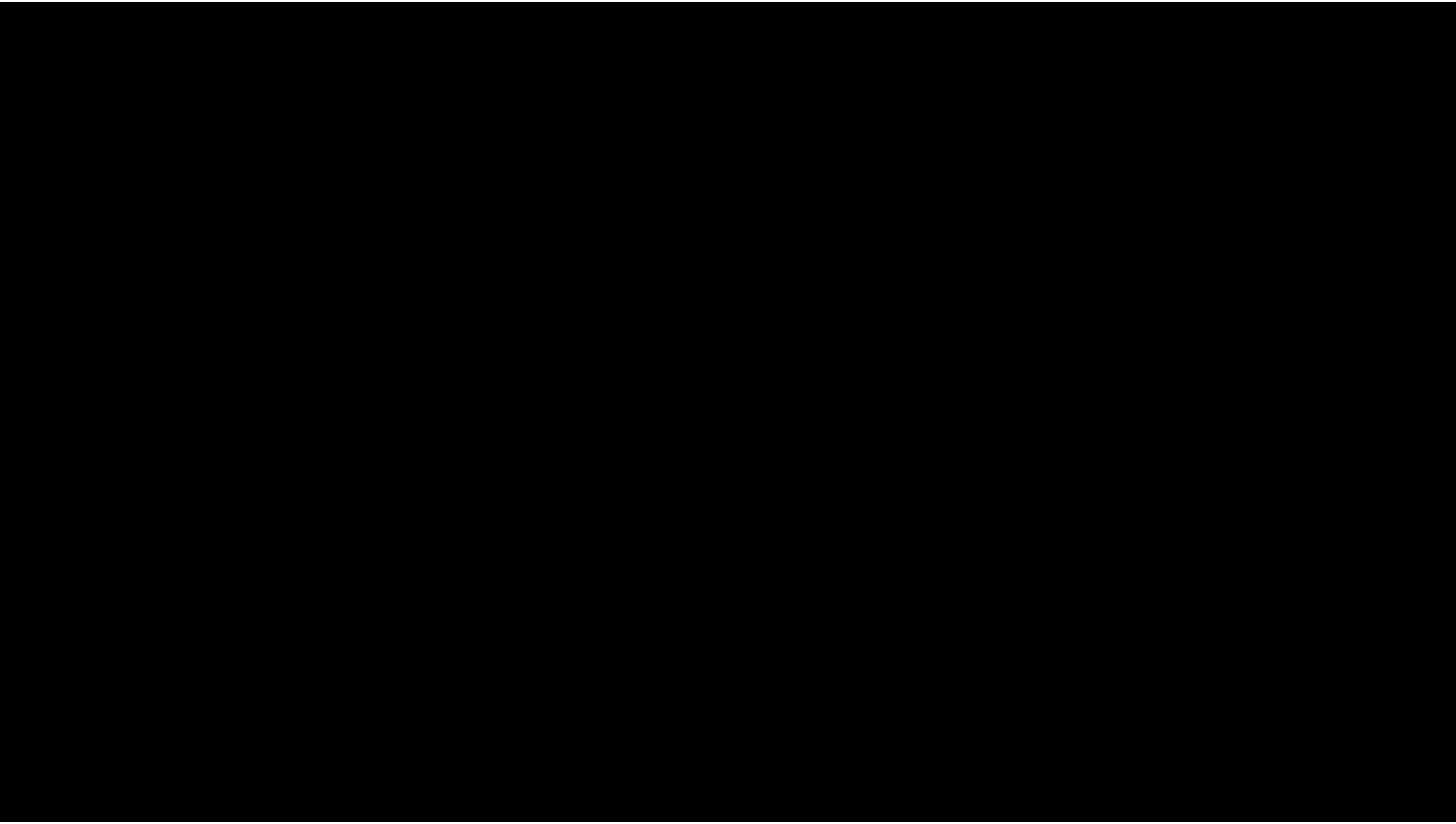
1. **Introduction to the lecture**
2. Design and Balancing mathematical method (Patterson & Albracht)

Introduction to the lecture



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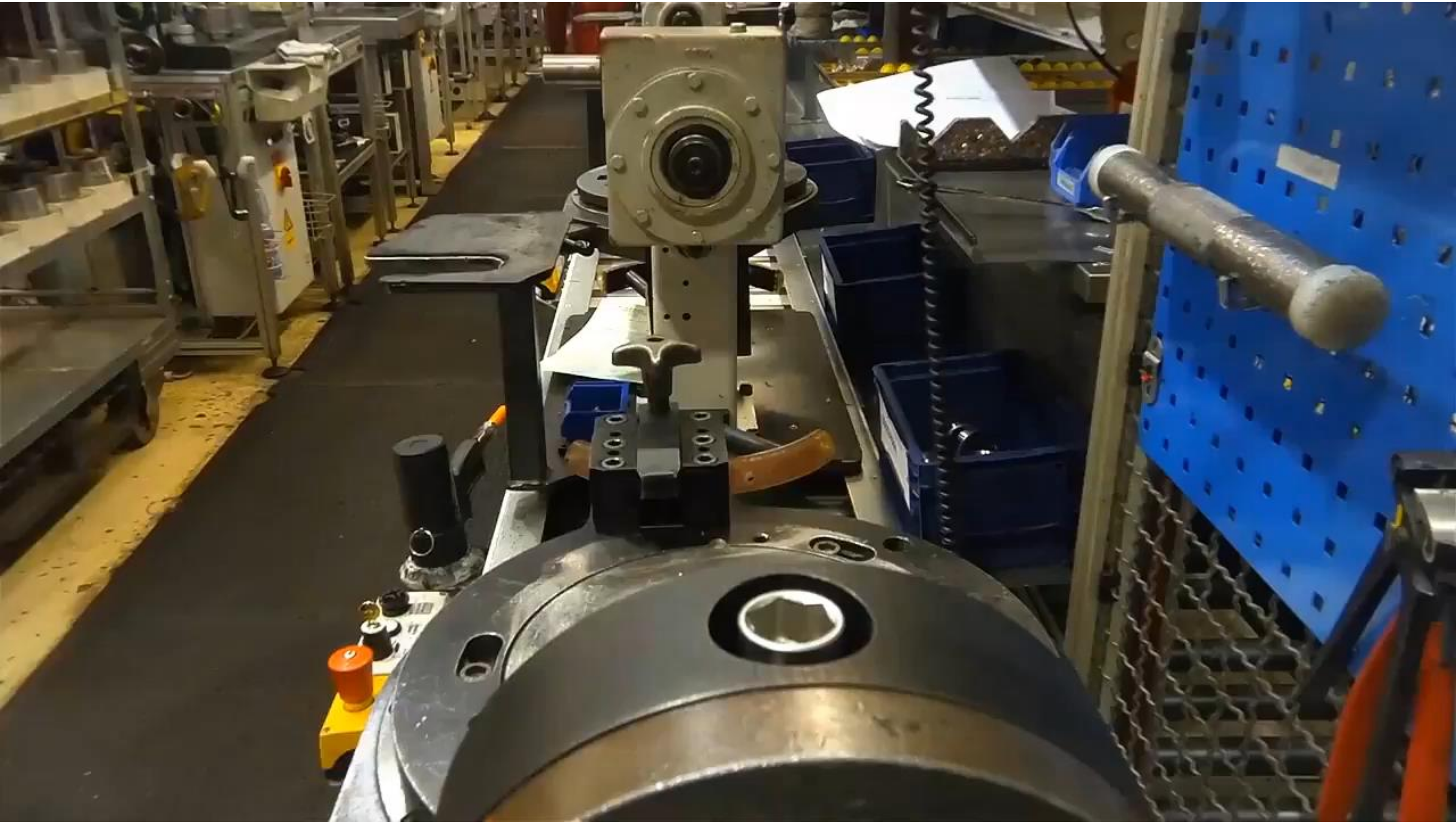
Introduction to the lecture



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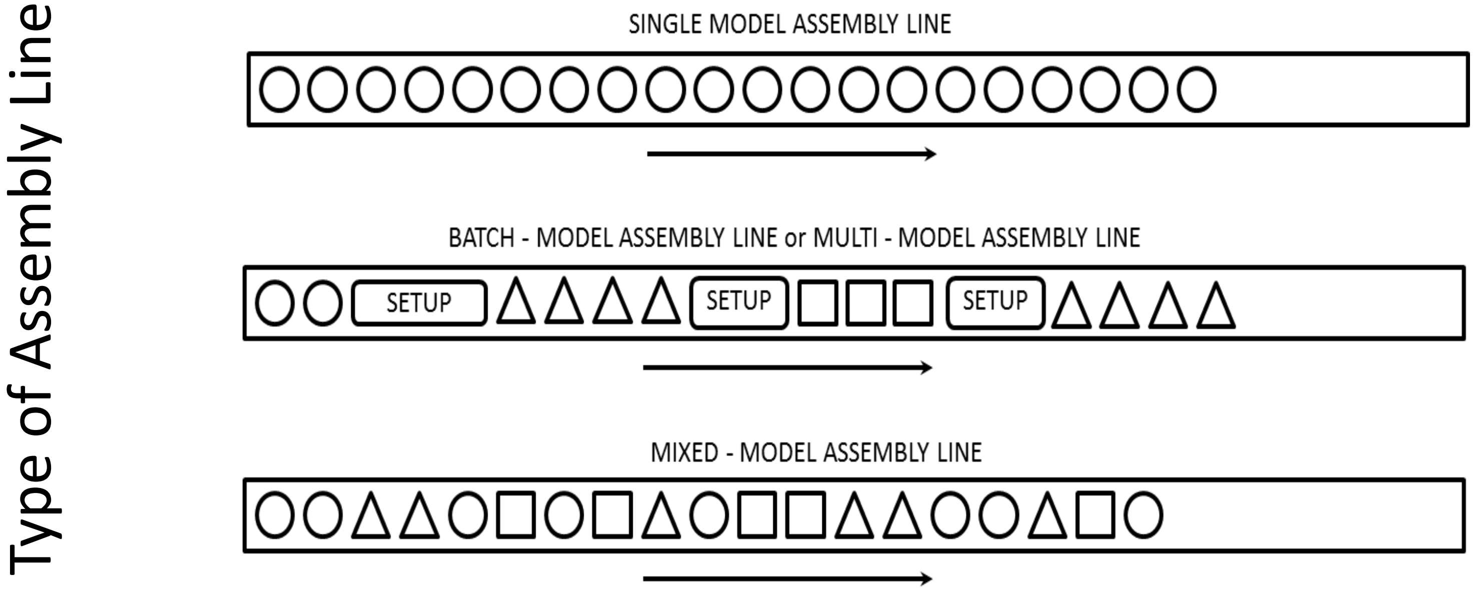
Introduction to the lecture



Important characteristics of Assembly systems:

- **Flexibility:** BE ABLE TO ASSEMBLE MORE THAN 1 MODEL PER TIME, AND BE RECONFIGURABLE OVER TIME (LIFETIME OF A SYSTEM >> LIFETIME OF A PRODUCT)
- **Elasticity:** BE ABLE TO ASSEMBLE AT DIFFERENT THROUGHPUT/DEMAND

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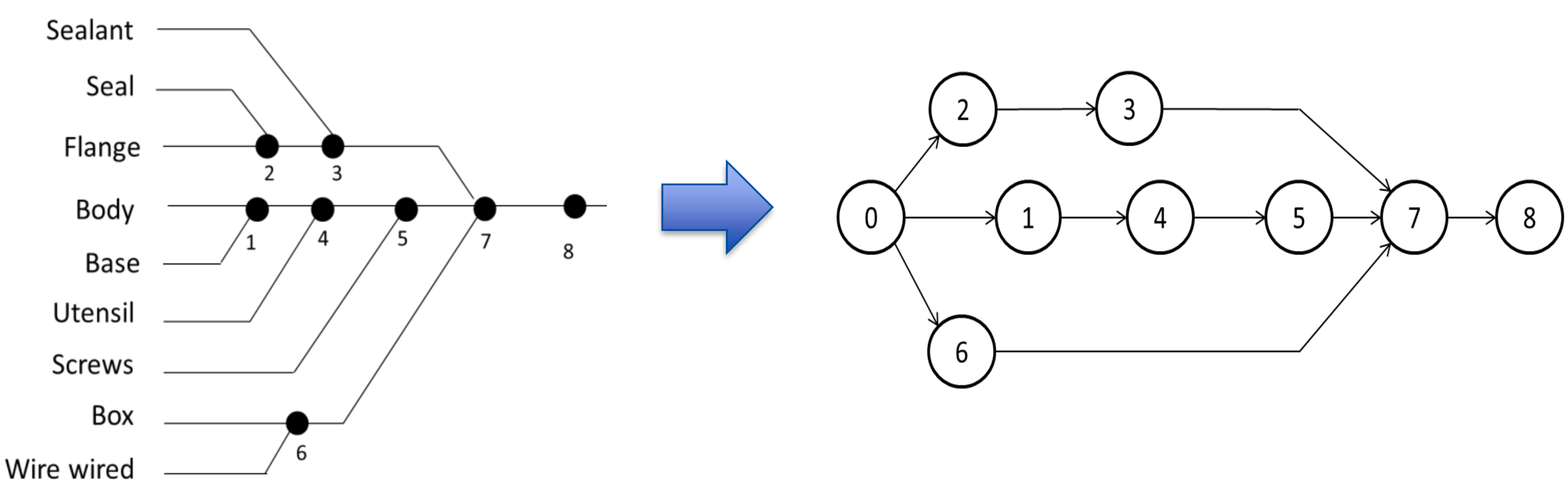
Introduction to the lecture

INPUT DATA AND INFO FOR THE DESIGN PHASE:

- Q = REQUIRED THROUGHPUT/DEMAND PER UNIT TIME [PCS/TIME]
- Takt Time (TT) = REQUIRED TIME BETWEEN TWO CONSECUTIVE PRODUCTS [TIME/PC] = AVAILABLE TIME/DEMAND
- PRECEDENCE DIAGRAM and TASK EXECUTION TIME (tj)
- GENERAL CONSTRAINTS (TOOL, EQUIPMENT, RESOURCES, ETC…)
- c = CICLE TIME [TIME/PC] = ACTUAL TIME BETWEEN TWO CONSECUTIVE PRODUCTS (DEPENDS ON THE DESIGN AND BALACING SOLUTION, BUT IT HAS TO BE SMALLER THAN TT)



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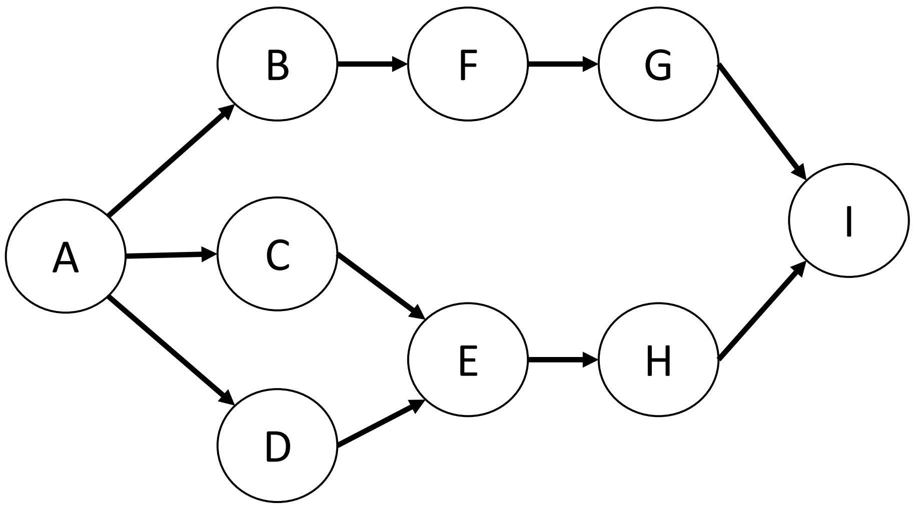

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A	30
B	40
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F	7
G	12
H	32
I	33

INPUT DATA AND INFO FOR THE DESIGN PHASE:

- Q = 60 pcs/h
- TT = 3600/Q = 60 s/pc

$m^* = \frac{\sum_j t_j}{TT}$ **minimum/theoretical** number of stations/resources

Introduction to the lecture

INPUT DATA AND INFO FOR THE DESIGN PHASE:

- Q = 60 pcs/h
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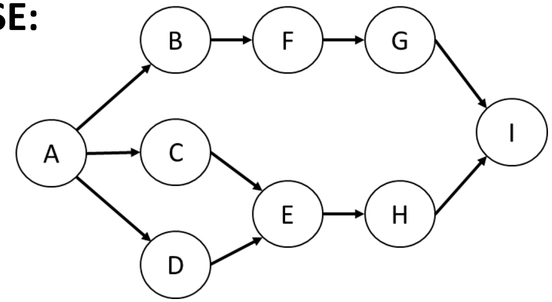
$$m^* = \frac{\sum_j t_j}{TT} = \frac{234}{60} = 3{,}9 \rightarrow 4\ minimum$$



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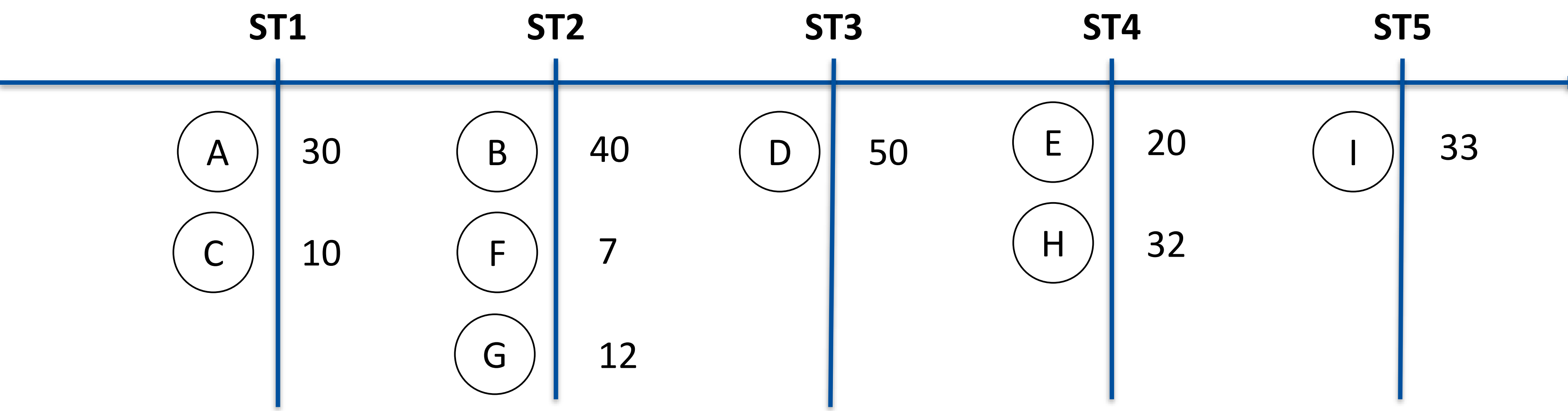

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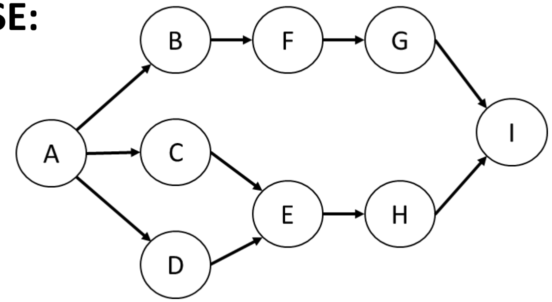

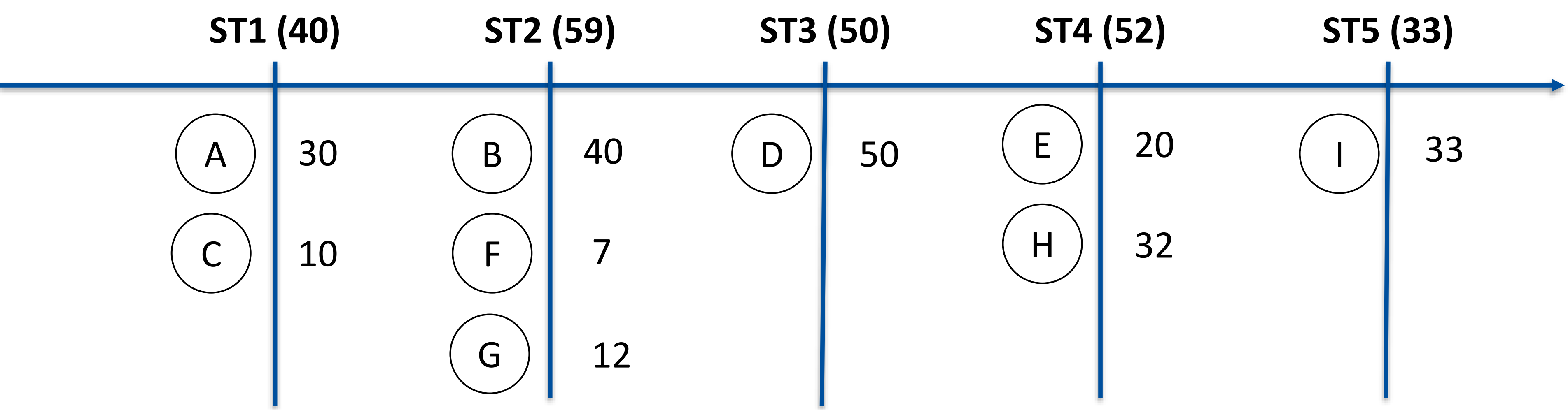

c min = minimum cycle time = 59 s -> max throughput = 3600/59 = 61 pcs/h

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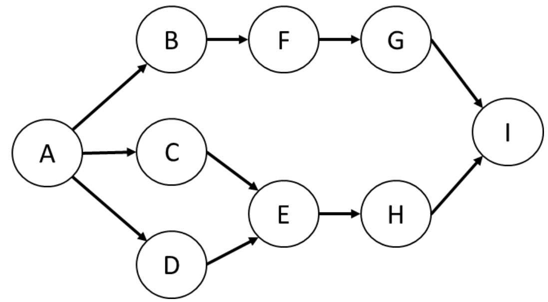
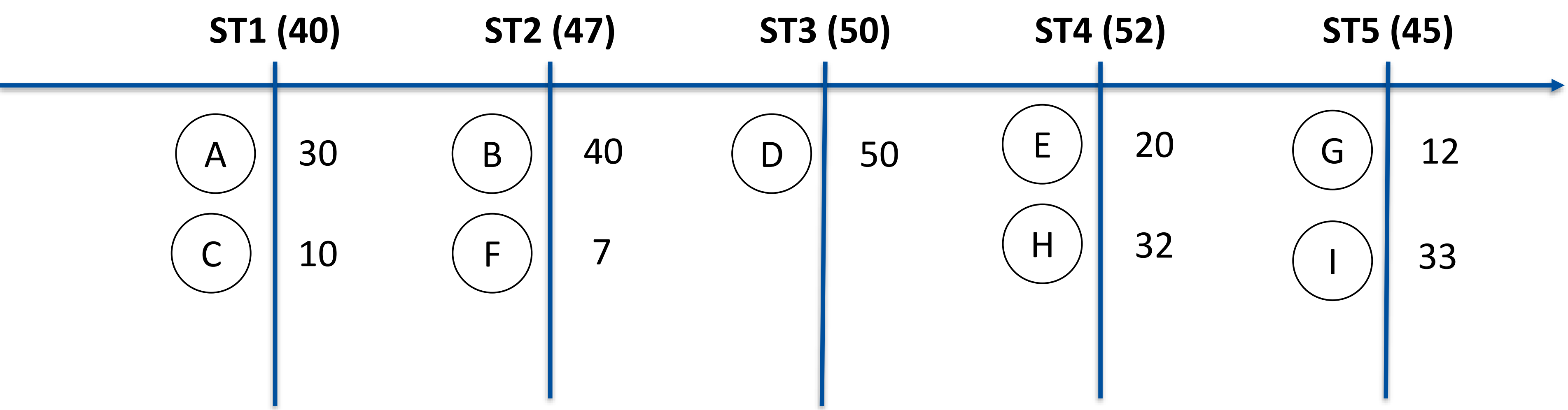
Alternative solution: move G to the ST5.





c min = minimum cycle time = 52 s -> max throughput = 3600/52 = 69,2 pcs/h (+8,2 pcs/h)

IT IS MORE PRODUCTIVE AND MORE BALANCED!!!

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Introduction to the lecture

Previous relevant steps:

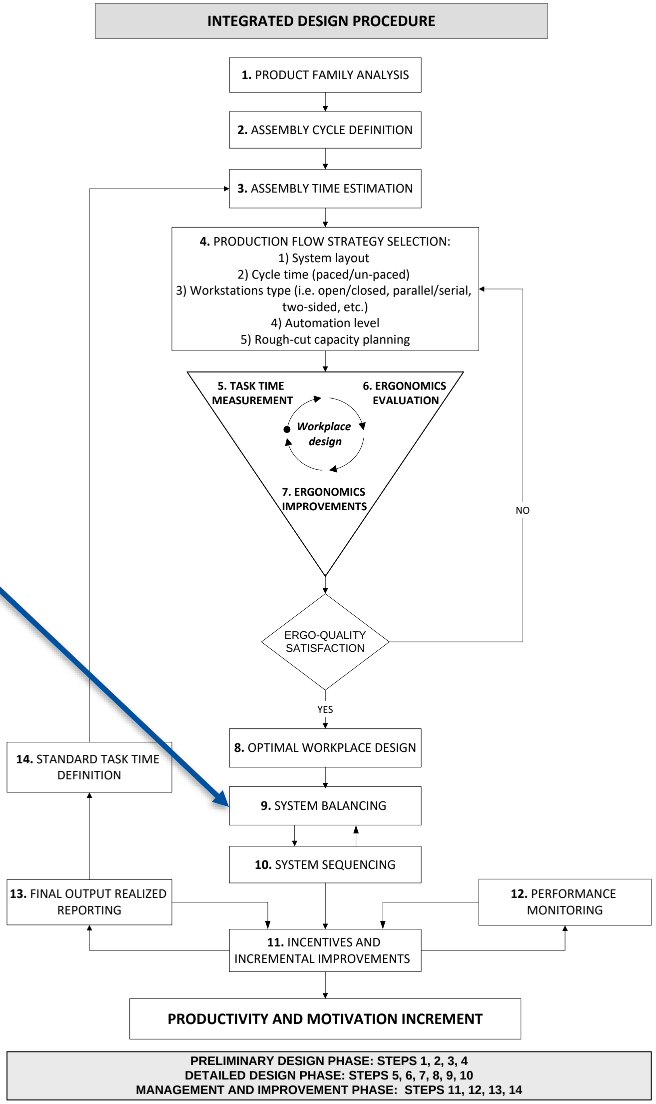
- Definition of the product family,
- Definition of the combined precedence diagram
- Definition of the required throughput = required cycle time
- Definition of the production flow strategy
- Definition of workplace design

-> we have:

- throughput, required cycle time/takt time, precedence diagram, layout

-> next step:

TO ASSIGN THE TASKS TO THE WORKSTATIONS/RESOURCES



Agenda

1. Introduction to the lecture
2. **Design and Balancing mathematical method (Patterson & Albracht)**

Design and Balancing

SINGLE-MODEL ASSEMBLY LINE BALANCING PROBLEM

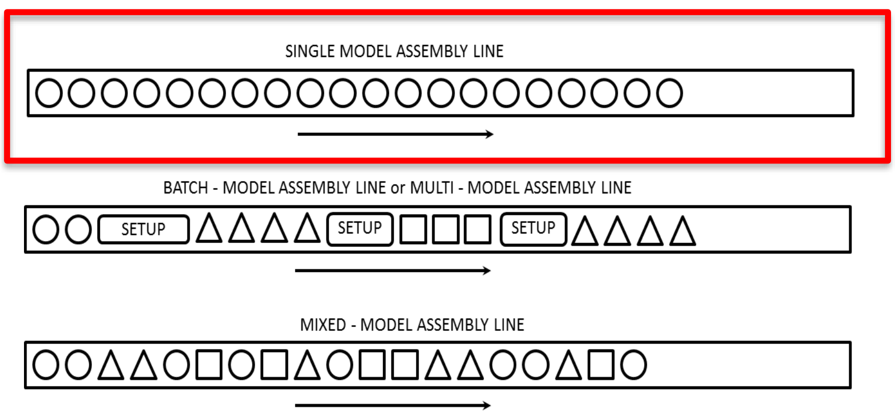
		Cycle time: Given	Cycle time: Minimize
N° Stations	Given	SALBP-F	SALBP-2
N° Stations	Minimize	SALBP-1	SALBP-E

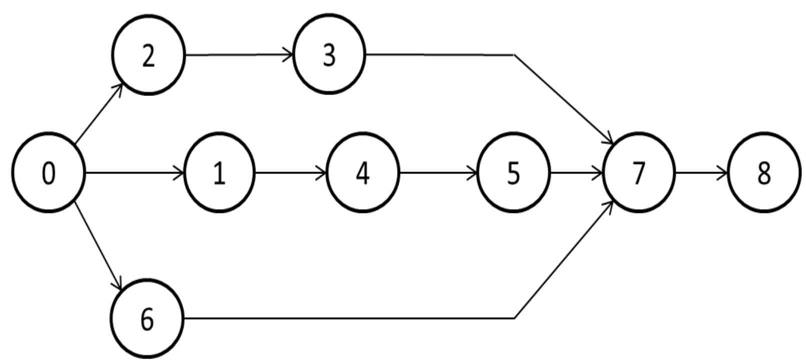
The SALBP-2 allows to minimize the cycle time for a defined number of stations, and it is used to increase the productivity of an existent assembly line.

Design and Balancing – Patterson & Albracht method

The following mathematical formulation is based on the Patterson and Albracht algorithm (1975), and considers the following assumptions:

- Single-product assembly line
- Fixed cycle time
- Deterministic and integral operation times
- No assignment restrictions beside the precedence contraints
- Serial line layout with one-sided stations
- All stations are equally equipped





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Design and Balancing – Patterson & Albracht method

Generally, starting from the data regarding the tasks operation time, the precedence diagram and the required throughput, the balancing procedure follows these phases:

1. calculation of required cycle time/takt time (c or TT)
2. definition of number of stations (m)
3. identification of earliest and latest stations
4. definition of constraints
5. optimization of solution

Design and Balancing – Patterson & Albracht method

INPUT DATA:

- THROUGHPUT Q
- TASK TIME t_i
- LIST OF ACTIVITIES AND PRECEDENCE DIAGRAM (S_i; P_i; W_j)

OUTPUT:

- Number of stations
- Assignment of tasks to the stations based on the binary variable x_{ij} as 1, if task i is assigned to station j and 0 otherwise
- Objective functions to optimize

STEP 1: calculation of required cycle time/takt time (c or TT)

c or TT = 3600/Q = [s/pc] if Q is expressed in pcs/h

STEP 2: definition of number of stations (m)

$m^* = \frac{\sum_j t_j}{c}$ **minimum/theoretical** number of stations/resources

$\mathrm{m} = \lceil m^* \rceil$ **minimum required** number of stations/resources

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Design and Balancing – Patterson & Albracht method

STEP 3: identification of earliest and latest stations – to reduce the number of variables x_{ij}

- earliest station E_i considers that a task i must not start before all its preceding tasks are performed

$$E_i = \left\lceil \frac{\left(t_i + \sum_{h \in P_i} t_h\right)}{c} \right\rceil$$

- latest station L_i delimits the latest station to which the same task i can be allocated based on its successors and the total number of stations

$$L_i = m + 1 - \left\lceil \frac{\left(t_i + \sum_{h \in S_i} t_h\right)}{c} \right\rceil$$

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Design and Balancing – Patterson & Albracht method

STEP 4: definition of constraints

- Occurrence constraints (ensuring that each task is assigned to exactly one stations in its interval.

$$\sum_{j=E_i}^{L_i} x_{ij} = 1 \qquad for\ i = 1, \dots, n$$

- Cycle time constraints (guaranteeing the respect of cycle time in each station)

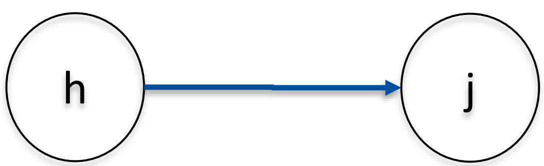
$$\sum_{i \in W_j} t_i \cdot x_{ij} \leq c \qquad for\ j = 1, \dots, m$$

Design and Balancing – Patterson & Albracht method

STEP 4: definition of constraints

- Precedence constraints (assuring that no task is assigned to an earlier station than its predecessor)

$$\sum_{j \in E_a}^{L_a} j \cdot x_{aj} \leq \sum_{k \in E_b}^{L_b} k \cdot x_{bk} \qquad for\ (a, b)\ and\ L_a \geq E_b$$



Eh = 1
Lh = 3
k = 1,2,3

Ej = 2
Lj = 4
k = 2,3,4

$$\underbrace{1 \cdot x_{h1} + 2 \cdot x_{h2} + 3 \cdot x_{h3}}_{\text{Station where h is assigned}} \leq \underbrace{2 \cdot x_{j2} + 3 \cdot x_{j3} + 4 \cdot x_{j4}}_{\text{Station where j is assigned}}$$

- Can do constraints (assuring that a task can be assigned to a station)

$$x_{ij} = 0 \text{ for i} \notin \ W_j$$

Design and Balancing – Patterson & Albracht method

STEP 5: optimization of solution

- Minimize the **smoothness index $\boldsymbol{SX}$** which measures the balance of workload among the stations as follows:

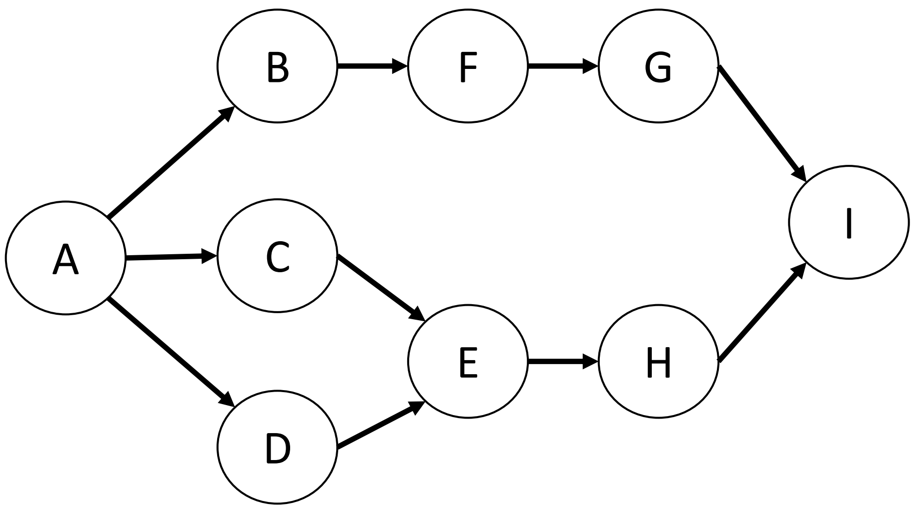
$$SX = \sum_{j=1}^{m} \left| \sum_{i \in W_j} t_i \cdot x_{ij} - \frac{\sum_i t_i}{m} \right|$$

- Minimize the Maximum Station Time, which measures the workload of the bottleneck of the system:

$$MST = \max_j \sum_{i \in W_j} t_i \cdot x_{ij}$$

Design and Balancing – Patterson & Albracht method

LET'S APPLY IT TO THE FIRST EXAMPLE



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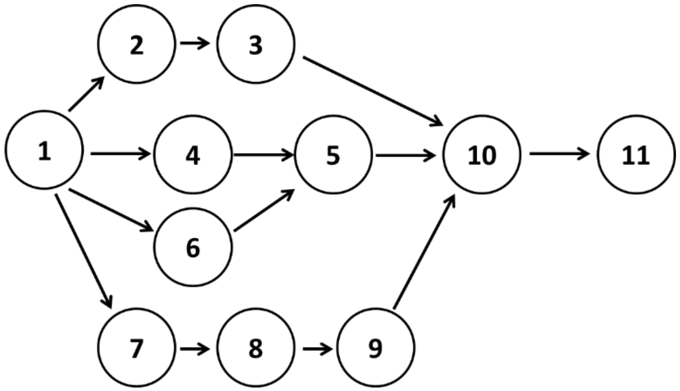
EXERCISE WITH EXCEL SOLVER

Design and balance an assembly system for the following product with the objective to have a total throughput of 28800 pcs/year, with 240 days/year and 2 shifts of 7.5 h/shift.

Solve the following questions:

- Design the system (n of stations).
- Balance the system.
- If each operator costs 30€/h, which will be the cost per product?

Task	Tj [min]
1	3
2	6
3	3
4	1
5	2
6	2
7	3
8	4
9	2
10	5
11	1



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