PK4450 - Lecture memo

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Introduction to maintenance optimization

Introduction

The main objective of this course is to increase our understanding of maintenance and improve maintenance modelling skills. In particular we focus on Data Driven Prognostics and Predictive Maintenance. With maintenance we understand "the combination of all technical and administrative actions, including supervision actions, intended to retain an item in, or restore to, a state in which it can perform a required function". With maintenance optimisation we understand "balancing the cost and benefit of maintenance". There are many aspects of maintenance optimisation, and some of these are:

- Deciding the amount of preventive maintenance (i.e. choosing maintenance intervals)
- Deciding the degradation level upon it is beneficial to replace a component
- Deciding whether to do first line maintenance (on the cite), or depot maintenance
- Choosing the right number of spare parts in stock
- Preparedness with respect to corrective maintenance
- Time of renewal
- Grouping of maintenance activities.

With preventive maintenance (PM) we understand "the maintenance carried out at predetermined intervals or according to prescribed criteria and intended to reduce the probability of failure or the degradation of the functioning of an item" (EN 13306). There exist several approaches to determine a preventive maintenance program. A concept that is becoming more and more popular is the concept of Reliability Centred Maintenance (RCM). RCM is “a systematic consideration of system functions, the way functions can fail, and a priority based consideration of safety and economics that identifies applicable and effective PM tasks.

An RCM analysis is usually conducted as a pure qualitative analysis with focus on identifying appropriate maintenance tasks. However, the RCM methodology does not give support for quantitative assessment in terms of e.g., interval optimisation. In this course we will present the framework for optimising maintenance interval as well.

The strength of RCM is its systematic approach to consider all system functions, and set up appropriate maintenance task for these functions. On the other hand, RCM is not a methodology that could be used to define a renewal strategy. To determine optimal renewal strategies for larger systems we usually work work with Life Cycle Cost modelling (LCC), see example in TPK5115.

In contrast to traditional calendar based preventive maintenance the main idea of a predicative maintenance strategy is to utilize component condition, future loads, and opportunity windows to determine a “just in time” plan for maintenance. Condition information is basically used for:

1. Anomaly detection, i.e., early warning of coming events
2. Diagnostics, i.e., the search for root causes behind symptoms observed
3. Prognostics, i.e., estimation of degradation rate, time to failure, remaining useful life etc based on relevant information

Classical maintenance optimization

Within maintenance optimisation literature it is common to present some basic models such as the Age Replacement Policy (ARP) model, the Block Replacement Model (BRP) and the Minimal Repair Policy (MRP). Such models were introduced by Barlow and Hunter (1960) and have later been generalised in several ways, see e.g. Block et. al. 1988, Aven and Bergman (1986), and Dekker (1992). There exists also several major (review) articles in this area, e.g. Pierskalla and Voelker (1979), Valdez Flores and Feldman (1989), Cho and Parlar (1991) and Wang (2002).

Some of these classical methods will be discussed in this course. However, in order to have a standardized framework for the modelling we will introduce a common term, i.e., the “effective failure rate” which may be applied in very many situations.

The effective failure rate is the expected number of failures per unit time as a function of our preventive maintenance strategy. In the simplest cases the preventive maintenance strategy is to maintain at predefined intervals. We will denote the failure rate by $\lambda_{\mathrm{E}}()$. For example if we as a preventive maintenance activity replace the item at intervals of fixed length τ, we write the effective failure rate as $\lambda_E(\tau)$.

Now there are two challenges, first we want to establish the relation $\lambda_E(\tau)$ depending on the (component) failure model we are working with, then next, we need to specify a cost model to optimise. The cost model will generally involve system models as fault tree analysis, Markov analysis etc. This enables us to find the optimum maintenance intervals in a two step procedure. Note also that when we use $\lambda_E(\tau)$ in the system models we then assume a "constant failure rate" which of course is an approximation for ageing components. However, if the component is maintained preventively it is reasonable that those failures "escaping" our maintenance strategy are independent of time, hence the constant failure rate approximation is reasonable.

Introductory example

Consider a component for which the effective failure rate is given by $\lambda_E(\tau) = \tau/100$, where τ is the maintenance interval. Assume that the cost of a component failure is $C_F = 10$ (corrective maintenance cost, production loss etc). Further let $C_{PM} = 1$ be the cost per preventive maintenance action carried out at intervals of length τ. The total cost per unit time is then given by:

$$C(\tau) = C_{PM}/\tau + C_F\tau/100 = 1/\tau + \tau/10 \tag{1}$$

To minimize cost we differentiate, and equate to zero:

$$\frac{dC(\tau)}{d\tau} = \frac{-1}{\tau^2} + \frac{1}{10} = 0 \Rightarrow \tau = \sqrt{10} \approx 3.16 \tag{2}$$

Expanding the cost model

In many situations we would be more explicit on the cost of a failure. A standard form of the cost model to consider is given by:

$$C(\tau) = C_{PM}/\tau + \lambda_E(\tau)C_F = C_{PM}/\tau + \lambda_E(\tau)[C_{CM} + C_{EP} + C_{ES} + C_{EM}] \tag{3}$$

where

- C_{PM} is the cost per preventive maintenance activity
- $C_{CM}$is the cost of a corrective maintenance activity
- C_{EP} is the expected economic value of production loss upon a failure often expressed as: $C_{EP} = \Pr(P)[C_P\text{MDT} + C_T]$, where
  - $\Pr(P)$ is the probability that a component failure gives a system failure with production loss
  - C_P is the value of production loss per time unit (typically per hour) when the system is down

- MDT is the mean down time after a failure (typically in hours)
- C_T a fixed cost upon a trip, i.e., when the system goes down independent of the duration of the downtime

* C_{ES} is the expected economic value related to safety loss upon a failure, and is often expressed as: $C_{ES} = \Pr(S)C_S$, where
  - $\Pr(S)$ is the probability that a component failure gives a system failure with safety impact
  - C_S is the corresponding cost given that the "safety event" occurs
* C_{EM} is the expected economical value of material losses upon the component failure .

By an explicit modelling of the failure cost C_P we might investigate other aspect of the optimization problem than the effective failure rate. For example MDT might depend on availability of spare parts, preparedness etc, further $\Pr(P)$ might depend on the reliability of backup systems, and $\Pr(S)$ might depend on other safety barriers. In the following we will not pursue this idea, and generally we collect all costs into C_F.

The effective failure rate, $\lambda_E()$

There is no general formula for the effective failure. We need to consider each situation individually. However, there are some standard situations where we are able to provide explicit formulas or ways to calculate the effective failure rate.

The "simple" situation

A very simple way to find the effective failure rate is described below. This approach is very often sufficient, although the approximation might be rather rough. Assume that we have an ageing item, i.e., an item with an increasing failure rate function $z(t)$. Further assume that times to failure are Weibull distributed with mean time to failure MTTF and ageing (shape) parameter α. If we replace the item periodically with times between replacements equal to τ, we approximate the effective failure rate with the average failure rate function in the interval $[0,\tau]$. This gives:

$$\lambda_E(\tau) = \left(\frac{\Gamma(1+1/\alpha)}{\text{MTTF}}\right)^{\alpha} \tau^{\alpha-1} \tag{4}$$

A slightly improved approximation

Equation (4) is not very accurate if the $\tau > \text{MTTF}/3$. It might be shown that a better approximation is given by:

$$\lambda_E(\tau) = \left(\frac{\Gamma(1+1/\alpha)}{\text{MTTF}}\right)^{\alpha} \tau^{\alpha-1}\gamma(\tau,\alpha,\text{MTTF}) \tag{5}$$

where the correction term $\gamma(\tau, \alpha, \text{MTTF})$ is given by:

$$\gamma(\tau, \alpha, \text{MTTF}) = \left[1 - \frac{0.1\alpha\tau^2}{\text{MTTF}^2} + \frac{(0.09\alpha - 0.2)\tau}{\text{MTTF}}\right] \tag{6}$$

Computationally Equation (5) will not cause any problems. But if we search for analytical solutions we will not be able to find such ones with this improved approximation.

An almost exact approximation

The effective failure rate is the expected number of failures per unit time. Assuming that the item always is replaced by a new one every τ time unit, the expected number of failures in one cycle of length τ is given by the *renewal function*, $W(\tau) = \text{E}(N(\tau))$. This means that the effective failure rate is given by:

$$\lambda_{\text{E}}(\tau) = \frac{W(\tau)}{\tau} \tag{7}$$

From the fundamental renewal equation, $W(t) = F_T(t) + \int_0^t W(t-x) f_T(x)dx$ we are able to set up an iterative scheme to calculate the effective failure rate. Assume we have a reasonable initial approximation, for $W(t)$, say $W_0(t)$. We may then use the following iteration scheme:

$$W_i(t) = F_T(t) + \int_0^t W_{i-1}(t-x) f_T(x)dx \tag{8}$$

to obtain better and better solutions for $W(t)$. An initial approximation would be:

$$W_0(t) = \lambda_{\text{E}}(t)t = \left(\frac{\Gamma(1+1/\alpha)}{\text{MTTF}}\right)^\alpha t^{\alpha-1}t = \left(\frac{\Gamma(1+1/\alpha)}{\text{MTTF}}\right)^\alpha t^\alpha \tag{9}$$

To solve the convolution integral in Equation (8) we need numerical methods. For each iteration we need to maintain a vector of W-values. $F_T(t)$ and $f_T(t)$ are the cumulative distribution function and probability density function for the time to failure respectively, and in our case we usually assume Weibull distributed times to failure. For typical values of $t < \text{MTTF}$ the solution converges after 2-3 iterations.

Although we need rather few iterations, computational time might still be long because it is required to calculate the effective failure rate several times in order to minimize total expected cost.

Exercise

Assume a component is replaced with a new one every τ time unit independent of when failures occur. Calculate the effective failure rate by Equation (8) when $\tau = \text{MTTF}/2$ and $\alpha = 3$. Compare with equations (4) and (5).