Problem 2.7

Table 1: Decision variables

Variable	Definition
x_1	Frozen fish fillet, today
x_2	Fresh fish fillet, today
x_3	Fish au gratin, today using high quality fish
x_4	Fish au gratin, today using low quality fish
x_5	Slack, high quality
x_6	Slack, low quality
x_7	Slack, workers

Table 2: Parameters

Parameter	Value	Description
p_{Fz}	20	Profit frozen fish fillet
p_{Fr}	40	Profit fresh fish fillet
p_{Fg}	10	Profit fish au gratin
u_{Fz}	0.7	Utilization frozen fish fillet
u_{Fr}	0.6	Utilization fresh fish fillet
u_{Fg}	0.8	Utilization fish au gratin
H_{HQ}	2 000	Holding high quality
H_{LQ}	3 000	Holding low quality
ρ_{Fz}	500	Production rate per worker, frozen fish fillet
ρ_{Fr}	750	Production rate per worker, fresh fish fillet
ρ_{Fg}	1 000	Production rate per worker, fish au gratin
n_W	5	Number of workers

Objective function:

Maximize: $Z = p_{Fz}u_{Fz}x_1 + p_{Fr}u_{Fr}x_2 + p_{Fg}u_{Fg}x_3 + +p_{Fg}u_{Fg}x_4 = 14x_1 + 24x_2 + 8x_3 + 8x_4$

$$\begin{aligned} \text{Subject to: } x_1 + x_2 + x_3 + x_5 &= H_{HQ} \\ x_4 + x_6 &= H_{LQ} \\ 1/\rho_{Fz}x_1 + 1/\rho_{Fr}x_2 1/\rho_{Fg}x_3 + x_6 &= n_W \end{aligned}$$

Variables:				x_1	x_2	x_3	x_4	x_5	x_6	x_7
Objective function c_i's				14	24	8	8	0	0	0
Variable values:		Z=	0							
Relative profit:										
Number of variables	7									
Number of constraints	3									
Constraints coefficients	Basic var.	Upper limit	b_i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	a_{i7}
	5		2000	1	1	1	0	1	0	0
	6		3000	0	0	0	1	0	1	0
	7		5	0.002	0.001333	0.001	0.001	0	0	1

Figure 1: Initial tableau

Calculation scheme for the risk priority number:
$\text{RP}_1 = c_1 - c_5 a_{1,1} - c_6 a_{2,1} - c_7 a_{3,1} = 14 - 0 \cdot 1 - 0 \cdot 0 - 0 \cdot 0.002 = 14$
$\text{RP}_2 = c_2 - c_5 a_{1,2} - c_6 a_{2,2} - c_7 a_{3,2} = 24 - 0 \cdot 1 - 0 \cdot 0 - 0 \cdot 1.333\text{E} - 03 = 24$
$\text{RP}_3 = c_3 - c_5 a_{1,3} - c_6 a_{2,3} - c_7 a_{3,3} = 8 - 0 \cdot 1 - 0 \cdot 0 - 0 \cdot 0.001 = 8$
$\text{RP}_4 = c_4 - c_5 a_{1,4} - c_6 a_{2,4} - c_7 a_{3,4} = 8 - 0 \cdot 0 - 0 \cdot 1 - 0 \cdot 0.001 = 8$

The idea is to repeat for all non-basic variables, e.g., for non-basic variable x_1: Increase it's value with one unit from 0 to 1. This increases profit by c_1. To balance constraint equation 1, basic variable x_5 has to be reduced by $a_{1,1}$ units. Profit is then reduced by $c_5 a_{1,1}$. Similarly to balance constraint equation 2, basic variable x_6 has to be reduced by $a_{2,1}$ units, with a profit reduction of $c_6 a_{2,1}$ and so on.

The calculation shows that variable x_2 has the highest risk priority number (24), and x_2 is therefore the new variable to add as basic variable.

Calculation scheme for the Upper Limit:
$\text{UL}_{\text{Eq1}} = b_1/a_{1,2} = 2000/1 = 2000$
$\text{UL}_{\text{Eq2}} = b_2/a_{2,2} = 3000/0 = \infty$
$\text{UL}_{\text{Eq3}} = b_3/a_{3,2} = 5/1.333\text{E} - 03 = 3750$

Idea: By increasing the new potential basic variable, i.e., x_2, the existing basic variables have to be reduced. But we can not reduce a basic variable so much that it becomes negative. For example in constraint equation 1 basic variable x_5 can be reduced from the current value b_1 to 0. This means that a maximum of b_1 units could be “released” and taken by the new basic variable, i.e., x_2. For each unit we increase x_2 we need to “eat” $a_{1,2}$ units, hence the upper limit is $b_1/a_{1,2}$. Similarly for the other basic variables. The basic variable with the *lowest* upper limit is the one that is “limiting” the new basic variable most, and is therefore removed as a basic variable.

Here equation 1 has the lowest value of the upper limit, hence x_2 replaces x_5 as a new basic variable. The first step of the pivot operation is to divide constraint equation 1 (corresponding to the basic variable to remove) by $a_{1,2}$. Then for each of the other constraint equations we need to “get rid of” $a_{i,2}$. In constraint equation 2 we do not have to do anything since $a_{i,2} = 0$. For constraint equation 3 we subtract constraint equation 1 multiplied by $a_{3,2}$ from constraint equation 3 to get $a_{3,2} = 0$. For example, $b_3 = b_3 - b_1 a_{3,2} = 5 - 2000 \cdot 0.00133 = 2.333$. The result after applying the pivot operation is shown in Figure 2:

Variables:				x_1	x_2	x_3	x_4	x_5	x_6	x_7
Objective function c_i's				14	24	8	8	0	0	0
Variable values:		Z=	48000	0	2000	0	0	0	3000	2.333
Relative profit:				14	24	8	8	0	0	0
Number of variables	7									
Number of constraints	3									
Constraints coefficients	Basic var.	Upper limit	b_i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	a_{i7}
	2	2000	2000	1	1	1	0	1	0	0
	6	1E+30	3000	0	0	0	1	0	1	0
	7	3750	2.33333	7E-04	0	-3E-04	0.001	-0	0	1

Figure 2: Tableau after the first pivot operation

In the second iteration x_4 has the highest risk priority number (8). Calculating the upper limit for the basic variables, equation 3 has the lowest value (2333.33), hence variable x_7 is to be replaced by x_4. Applying the pivot operation we get the result shown in Figure 3:

In the third iteration all the risk priority numbers are negative, hence the solution cannot be improved. Resolving the constraints equations gives $x_2 = 2\ 000$ and $x_2 = 2\ 333$. Total profit is $Z = 66\ 666$ We note that $x_6 = 667$, hence not all raw material is used. If more workers were available we would have been able to use all the raw material.

Variables:				x_1	x_2	x_3	x_4	x_5	x_6	x_7
Objective function c_i's				14	24	8	8	0	0	0
Variable values:			Z= 66666.7	0	2000	0	2333.3	0	666.7	0
Relative profit:				-10	0	-16	8	-24	0	0
Number of variables	7									
Number of constraints	3									
Constraints coefficients	Basic var.	Upper limit	b_i	a_{i1}	a_{i2}	a_{i3}	a_{i4}	a_{i5}	a_{i6}	a_{i7}
	2	1E+30	2000	1	1	1	0	1	0	0
	6	3000	666.667	-0.67	0	0.3333	0	1.3	1	-1000
	4	2333.3333	2333.33	0.667	0	-0.333	1	-1	0	1000

Figure 3: Tableau after the second pivot operation