Problem 4.5

The Markov transition matrix is given by:

$$\mathbf{A} = \begin{bmatrix} -\lambda & \lambda & 0 \\ \mu & -\lambda-\mu & \lambda \\ 0 & -2\mu & 2\mu \end{bmatrix} = \begin{bmatrix} -0.05 & 0.05 & 0 \\ 0.067 & -0.117 & 0.05 \\ 0 & 0.133 & -0.133 \end{bmatrix}$$

The matrix to use in the iterative scheme is:

$$\mathbf{A}\Delta t + \mathbf{I} = \begin{bmatrix} 0.75 & .25 & 0 \\ 0.333 & 0.417 & 0.25 \\ 0 & 0.667 & 0.333 \end{bmatrix}$$

The iterative scheme to use is:

$$\mathbf{P}(t+\Delta t) \approx \mathbf{P}(t)[\mathbf{A}\Delta t + \mathbf{I}]$$

Giving

t	$P_0(t)$	$P_2(t)$	$P_2(t)$
0	1	0	0
5	0.75	0.25	0
10	0.646	0.292	0.063
15	0.582	0.325	0.094
20	0.544	0.343	0.112
25	0.523	0.354	0.123
30	0.510	0.360	0.130
35	0.503	0.364	0.133
40	0.498	0.366	0.135
45	0.496	0.367	0.137
50	0.494	0.368	0.137
55	0.494	0.369	0.138
60	0.493	0.369	0.138

After half an hour the solution is close to the steady state solution:

Table 1: Steady state solution

j	P_j
0	0.4927
1	0.369
2	0.138